

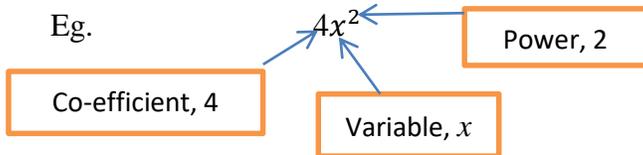
STRAND : ALGEBRA

1. Algebraic Expression

Recalling:

- Labeling of an Algebraic term

Eg.



- Like terms** : Are more than one terms that have the same variable and same power.

<p>Eg.1</p> $x + 3x$	<p>This expression has two terms. x and $3x$ Are they like terms? (see above the definition of like terms) Term 1 is x: variable is x and the power is 1 Term 2 is $3x$: variable is x and the power is 1.</p> <p>\therefore Since these two terms have the same variable, x and same power, 1 they are like terms.</p>
<p>Eg2.</p> $10x^2 - 4x^2$	<p>What about these terms? Term 1 is $10x^2$: variable is x and the power is 2. Term 2 is $4x^2$: variable is x and the power is 2.</p> <p>Yes, they are like terms : have the same variable , x and same power, 2.</p>

** Take notice of these two eggs. The coefficient has no saying in determining of the like terms.

<p>Eg.3</p> $x + 3x^2$	<p>This expression has two terms. x and $3x^2$ Are they like terms</p> <p>Term 1 is x: variable is x and the power is 1 Term 2 is $3x^2$: variable is x and the power is 2.</p> <p>\therefore Since these two terms have the same variable, <i>but</i> different power, they are NOT like terms or known as Unlike terms.</p>
<p>Eg4.</p> $10y^2 - 4x^2$	<p>What about these terms? Term 1 is $10y^2$: variable is y and the power is 2. Term 2 is $4x^2$: variable is x and the power is 2.</p> <p>$10y^2$ and $4x^2$ are Unlike terms. Why? They have different variable.</p>

Self – check Exercise 1:

Which of these expressions have like terms?

- $5x + 4x$
- $x^2 - y^2$
- $x + 2x - x^2$
- $a - 3a + 5a$
- $9b + 2b$
- $m + 3m^2$

Simplifying

- You can simplify algebraic expressions by addition and subtraction only if the terms are Like. (ie. You Can't Add or Subtract Unlike terms)

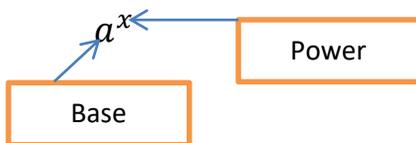
With the above examples.

Eg.1 $x + 3x$	Since the terms in this expression are Like terms you can add them. How to add/subtract like terms. - Add/subtract the coefficients of the terms and state the same variable. Ie. $x + 3x = (1 + 3)x = 4x$
Eg2. $10x^2 - 4x^2$	$10x^2 - 4x^2 = (10 - 4)x^2 = 6x^2$
Eg.3 $x + 3x^2$	This expression has unlike terms. Unlike terms CANNOT be added so $x + 3x^2 = x + 3x^2$
Eg4. $10y^2 - 4x^2$	Has unlike terms. $10y^2 - 4x^2 = 10y^2 - 4x^2$

Self – check Exercise 2:

- i. Simplify the following into its simplest form.
- | | |
|------------------|------------------------|
| a. $4x - 3x$ | f. $x + 2x - 2x^2$ |
| b. $n - 5n$ | g. $m^3 - 2m^2 + 7m^2$ |
| c. $4x^2 - 2x^2$ | |
| d. $2y + x$ | |
| e. $3y^2 + 3y^3$ | |

Recalling Law of Indices



1 $a^m \times a^n$	When terms have the same base – then add the power when multiply	$a^m \times a^n = a^{m+n}$ Eg. $y^2 \times y^4 = y^{2+4} = y^6$ $3^4 \times 3^5 = 3^{4+5} = 3^9$
$a^m \div a^n$ OR $\frac{a^m}{a^n}$	When terms have the same base – then subtract the power when divide	$\frac{a^m}{a^n} = a^{m-n}$ $\frac{a^5}{a^3} = a^{5-3} = a^2$
$(a^m)^n$	Raising a power to another power - Multiply the powers	$(a^m)^n = a^{m \times n}$ Eg. $(x^2)^4 = x^{2 \times 4} = x^8$ $(2^2)^2 = x^{2 \times 2} = 2^4$

Eg.1 $x \times 3x$	$x \times 3x = 3x^{1+1} = 3x^2$
Eg2. $\frac{10x^3}{5x^2}$ OR $\frac{10x^3}{5x^2}$	$\frac{10x^3}{5x^2} = \frac{10}{5} x^{3-2} = 2x$
Eg.3 $4ab^2 \times 5a^2b^3$	$4ab^2 \times 5a^2b^3 = (4 \times 5)a^{1+2}b^{2+3} = 20a^3b^5$
Eg4. $\frac{4x^4y^2}{8xy}$	$\frac{4}{8} x^{4-1}y^{2-1} = \frac{1}{2} x^3y$ OR $\frac{x^3y}{2}$

Self – check Exercise 3:

Simplify to its simplest form

- | | |
|---|--------------------------------|
| 1. $4^2 \times 4^3$ | 5. $4xy^2 \times 3xy^3$ |
| 2. $\frac{24m^6}{18m^2}$ | 6. $(x^2)^5$ |
| 3. $(2y^4)^2$ | 7. $\frac{(a^2b^3)^4}{a^4b^5}$ |
| 4. $\frac{16xy^3 \times 5x^2y}{20x^2y^3}$ | |

You can simplify any terms (Like or Unlike) by Multiplication / Division.

Eg.1 Simplify $2a \times 3b = (2 \times 3)ab = 6ab$	2a and 3b are Unlike terms. They have different variable, a and b. Can you simplify Unlike terms with multiplication ? Yes How?
Eg2. Simplify $4x^4 \div 2x^3$ or $\frac{4x^4}{2x^3} = \left(\frac{4}{2}\right)x^{4-3} = 2x$	$4x^2$ and $2x^3$ are unlike terms You can simplify with division

Self – check Exercise 4:

Simplify to its simplest form

- | | |
|--------------------------|-----------------------------------|
| 1. $8b \times 3c$ | 6. $m + m + m + m$ |
| 2. $8b + 3c$ | 7. $m \times m \times m \times m$ |
| 3. $5a \times 3a$ | 8. $3x^2 + x^4 - 2x^4$ |
| 4. $5a - 3a$ | 9. $5n \times 2mn^3$ |
| 5. $x^2 - x + 3x^2 - 2x$ | 10. $4p \times 2q \times 2r$ |

Expand and Simplify

1. Using distributive law

Expand	Solution
Eg.1 $7(y + 3x)$	$= 7 \times y + 7 \times 3x$ $= 7y + 21x$
Eg.2. $2x(x - 3x^2)$	$= 2x \times x + 2x \times 3x^2$ $= 2x^2 + 6x^3$
Expand and Simplify Eg.3 $5x(3 - x) - 3(2x - 7)$	$= 15x - 5x^2 - 6x + 21$ $= 15x - 6x - 5x^2 + 21$ $= 9x - 5x^2 + 21$
Eg.4. $2(a - 3b + c) + (a + 6b - c)$	$= 2a - 6b + 2c + a + 6b - c$ $= 2a + a - 6b + 6b + 2c - c$ $= 3a + c$

Self – check Exercise 5:

- | | |
|---|-----------------------------|
| 1. $3(x - 1)$ | 7. $5(3x - 2) + 6(x - 3)$ |
| 2. $-2(m + 3)$ | 8. $2(1 - 3x) - 4(x + 2)$ |
| 3. $5(2y + x)$ | 9. $x(3x + 2) - x(1 - 4x)$ |
| 4. $-(m - n)$ | 10. $2x(x + 1) - 3x(x + 5)$ |
| 5. $-3y(y - 4)$ | |
| 6. $4x(y + 3)$ | |
| 11. $2x(3x - 4) + x(1 - 4x) - 2x(4x + 3)$ | |
| 12. $-6(x + 7) + x(9x + 10) - (4 - x)$ | |
| 13. $-x(x + 5) + 10(x^2 + 2x - 1) - x(9x + 15)$ | |

2. Quadratic Expansions

A quadratic expression in x is one that can be expressed as $ax^2 + bx + c$ where $a \neq 0$

In a quadratic expression the highest power of the variable is 2.

Eg. $x^2 - 5x + 7$

when two linear expressions like $(x + 2)$ are multiply $(x + 1)$ together a quadratic is the result.

Examples

Expand and simplify	Solution
Eg.1 $(x + 2)(x + 1)$	$= x(x + 1) + 2(x + 1)$ $= x^2 + x + 2x + 2$ $= x^2 + 3x + 2$
Eg.2. $(x - 1)(x + 8)$	$= x(x + 8) - 1(x + 8)$ $= x^2 + 8x - x - 8$ $= x^2 + 7x - 8$
Eg.3 $(6 - x)(2x + 3)$	$= 6(2x + 3) - x(2x + 3)$ $= 12x + 18 - 2x^2 - 3x$ $= 12x - 3x + 18 - 2x^2$ $= 9x + 18 - 2x^2$
Eg.4. $(4x - 1)(3x - 2)$	$= 4x(3x - 2) - 1(3x - 2)$ $= 12x^2 - 8x - 3x + 2$ $= 12x^2 - 11x + 2$

Self – check Exercise 6:

Expand and Simplify these quadratic expressions:

- | | |
|----------------------|-------------------------|
| 1. $(x + 5)(x + 8)$ | 11. $(2x + 1)(x + 5)$ |
| 2. $(x + 9)(x + 7)$ | 12. $(x + 2)(3x + 5)$ |
| 3. $(x - 6)(x - 2)$ | 13. $(2x - 3)(x - 8)$ |
| 4. $(x - 12)(x - 3)$ | 14. $(x - 7)(5x - 2)$ |
| 5. $(x + 7)(x - 4)$ | 15. $(2x - 3)(3x - 4)$ |
| 6. $(x - 9)(x + 3)$ | 16. $(5x + 1)(2x + 7)$ |
| 7. $(2 + x)(x + 5)$ | 17. $(3x - 7)(2x + 5)$ |
| 8. $(x + 7)(8 + x)$ | 18. $(7x + 13)(5x - 4)$ |
| 9. $(9 - x)(x + 4)$ | |
| 10. $(1 - x)(5 - x)$ | |

Two special Quadratic Cases

1. Perfect Square

When both brackets in a quadratic expression are the same, we say that the expression can be written in **perfect square form**. Instead of repeating the same bracket more than once, we write the expression as a power.

Eg. $(x + 2)(x + 2)$ is better written as $(x + 2)^2$
 $(3x - 1)(3x - 1)$ is written as $(3x - 1)^2$

Perfect square can be expanded in exactly the same way as other quadratics.

<p>Expand and simplify Eg.1</p> $(x + 2)^2$	<p>Solution</p> $= (x + 2)(x + 2)$ $= x^2 + 2x + 2x + 4$ $= x^2 + 4x + 4$
<p>Eg.2.</p> $(3x - 1)^2$	$= (3x - 1)(3x - 1)$ $= 9x^2 - 3x - 3x + 1$ $= 9x^2 - 6x + 1$

Common Mistake :

Do NOT ever do something like $(a + b)^2 = a^2 + b^2$ (ie. $(a + b)^2 \neq a^2 + b^2$) because you have missed the point that $(a + b)^2 = (a + b)(a + b)$, which will give you as above $(a + b)^2 = a^2 + 2ab + b^2$

This is the shortcut of expanding a perfect square.

<p>Expand and simplify Eg.1</p> $(x + 2)^2$ $(a + b)^2 = a^2 + 2ab + b^2$	<p>Solution</p> $a = x$, and $b = 2$ $= a^2 + 2ab + b^2$ $= x^2 + 2(x)(2) + 2^2$ $= x^2 + 4x + 4$
<p>Eg.2.</p> $(3x - 1)^2$ $(a - b)^2 = a^2 - 2ab + b^2$	$a = 3x$, and $b = 1$ $= a^2 - 2ab + b^2$ $= (3x)^2 - 2(3x)(1) + 1^2$ $= 9x^2 - 6x + 1$

<p>Expand and simplify Eg.3</p> $(2x + 5)^2$ $(a + b)^2 = a^2 + 2ab + b^2$	<p>Solution</p> $a = 2x$, and $b = 5$ $= a^2 + 2ab + b^2$ $= (2x)^2 + 2(2x)(5) + 5^2$ $= 4x^2 + 20x + 25$
<p>Eg.4.</p> $(6 - 7x)^2$ $(a - b)^2 = a^2 - 2ab + b^2$	$a = 6$, and $b = 7x$ $= a^2 - 2ab + b^2$ $= (6)^2 - 2(6)(7x) + (7x)^2$ $= 36 - 84x + 49x^2$

Self – check Exercise 7:

1. $(x + 12)^2$
2. $(3 + 2x)^2$
3. $(5x + y)^2$
4. $(y - 8x)^2$
5. $(100x - 1)^2$
6. $(x^2 - 3)^2$
7. $(4x^2 + 5y^2)^2$

2. Difference of TWO squares

When the expressions in both brackets in a Quadratic expression are the same except for different signs $(x + 4)(x - 4)$, the resulting expression is called a **difference of two squares**.

$$(a + b)(a - b) = a^2 + ab - ab - b^2$$

$$= a^2 + ab - ab - b^2$$

$$\therefore (a + b)(a - b) = a^2 - b^2$$

Examples:

<p>Expand and simplify Eg.1</p> $(x - 4)(x + 4)$ $(a + b)(a - b) = a^2 - b^2$	<p>Solution</p> $a = x$, and $b = 4$ $= a^2 - b^2$ $= x^2 - 4^2$ $= x^2 - 16$
<p>Eg.2.</p> $(5 - 2x)(5 + 2x)$	$a = 5$, and $b = 2x$ $= a^2 - b^2$ $= 5^2 - (2x)^2$ $= 25 - 4x^2$

Self – check Exercise 8:

Expand and Simplify the following expression

1. $(x + 7)(x - 7)$
2. $(1 - x)(1 + x)$
3. $(7x - 5y)(7x + 5y)$
4. $(3x^2 + 5)(3x^2 - 5)$
5. $(x + 11)(x - 11)$
6. $(x^4 - 3)(x^4 + 3)$

Sum-UP Exercise:

Expand and Simplify

1. $(x - 3)(x + 2) - x(x + 7)$
2. $x(x - 1) + 2x(x + 7)$
3. $6(x + 3) + x(x + 7)$
4. $2x(3x - 1) + (x - 6)(x + 8)$
5. $(x + 2)(x - 3) + (2x + 1)(3x - 5)$
6. $(3x - 1)(x + 5) + (2x - 5)(x - 6)$
7. $(3x - 2)(5x + 3) - (x - 2)(7x - 1)$
8. $6(x - 1)((x+2) - (1 - x)(2 + x))$
9. $4x(2x + 1) - (x + 3)$
10. $4(x + 2)^2 + x(4x + 3) - (x + 5)(x - 1)$
11. $5x(x + 2) - (x + 2)^2 + 4(x + 2)(x - 2)$
12. $(3x - 5)^2 + (x - 4)^2 - 10(x + 4)(x - 1)$

Notes:

We have the solutions for all the Exercise. Call us to give you the solutions when you complete the exercise.

If you have a questions you can all us on

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