Fanau: Malo 'etau ma'u e 'aho ni.

Kataki Ko e veesi folofola ena ke kamata 'aki 'etau Kalasi. Lau ia, pea fai ha'o lotu pea toki kamata ho'o ngaahi ngaue:

Matiu 6: 33

" *Ka mou kumi mu'a ki hono pule'anga mo e ma'oni'oni 'a'ana pea 'e 'atu mo ia foki 'a e ngaahi me'a ko ia kotoa"*

STRAND : ALGEBRA (Notes #2)

Continue : Quadratics functions

Reminder: Quadratics functions are expression where the highest power is 2.

Form of Equation: $ax^2 \pm bx \pm c$, where $a \neq 0$

Eg. $x^2 + 2x + 1$, $x^2 - 2x$ $x^2 - 1$

We will continue on to Quadratic functions:

1. Factorization of Quadratic functions:

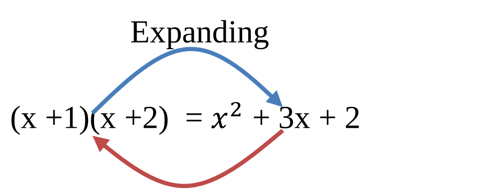
Factorising is the reverse/opposite process to expanding Brackets is where a sum is changed to a product involving brackets.

Remember the expanding; where product is changed to sum.

Product = sum

Eg. Expanding (x +1)(x +2) = $x^2 + 3x + 2$

Factorising is the opposite process of Expanding



(ie. Change from a sum to a product)

Eg. Factorise $x^2 + 3x + 2$

Sum = Product

$x^2 + 3x + 2 = (x+1)(x+2)$

Let us start Factorising with the TWO Special Cases of Quadratics:

1. Perfect Square
2. Difference of TWO Squares

1. Perfect square

When the two brackets in expression are the same.

Eg.1

a. (x+3)(x+3) which is written as $(x+3)^2$

$$(x+3)^2 = x^2 + 2(x)(3) + 3^2$$
$$= x^2 + 6x + 9$$

'E founga fefe ho'o 'ilo'i ko e **Perfect Square** ha Quadratic expression? ??

Notation: $ax^2 + bx + c$, where $a \neq 0$

PART 1: When a = 1 then we have $x^2 + bx + c$

Vakai ki he eg. ko ena 'i 'olunga tau 'ilo mei ai ko e **Perfect Square** 'a e $x^2 + 6x + 9$

How do you check for a perfect square?

Compare: $x^2 + 6x + 9$ to $x^2 + bx + c$

- Half the value of b.
- Then square the half of b.

∴ if squaring half of b = c, then the quadratic is a Perfect square.

ie. If $(\frac{1}{2}(b))^2 = c$

For eg. $x^2 + 6x + 9$: $(\frac{1}{2}(6))^2 = (3)^2 = 9$

∴Factorising of $x^2 + 6x + 9 = (x + \frac{1}{2}b)^2$

b = 6 , $\frac{1}{2}(6) = 3$ $= \underline{(x+3)^2}$

More examples on Factorising Perfect Square

Factorise:	Check.
Eg. 1 $x^2 + 8x + 16$ Is it a Perfect Square? Factorise	Half of 8 : $\frac{1}{2}(8) = 4$ Square of 4: $(4)^2 = 16$ Yes $x^2 + 8x + 16 = (x+4)^2$
Eg 2. $x^2 + 10x + 25$ Is it a perfect square? Yes	$\frac{1}{2}(10) = 5$ $(5)^2 = 25$ $x^2 + 10x + 25 = (x+5)^2$
Eg. 3 $x^2 - 20x + 100$ Is it a perfect square? Yes	$\frac{1}{2}(20) = 10$ $(10)^2 = 100$ $x^2 - 20x + 100 = (x-10)^2$
Eg. 4 $x^2 - 16x + 64$	$\frac{1}{2}(16) = 8$ $(8)^2 = 64$ $x^2 - 16x + 64 = (x-8)^2$

(Manatu'i 'oku toki ngaue 'aki pe founga ko eni if the quadratics is a Perfect Square :

Self-Check Exercise 2.1:

1. Check whether these quadratics are **Perfect Square.**
2. Factorise the quadratics that are Perfect Squares.

a. $x^2 + 2x + 1$	f. $x^2 + 14x + 49$
b. $x^2 + 3x + 2$	g. $x^2 + 22x + 121$
c. $x^2 - 4x + 4$	h. $x^2 - 6x + 7$
d. $x^2 - x + 5$	i. $x^2 - 24x + 144$
e. $x^2 - 12x + 36$	j. $x^2 + x + 1$

PART 2: When a > 1 then we have $ax^2 + bx + c$

Example: $\mathbf{4x^2 + 12x + 9}$

Check for Perfect Square : Find

i. Square root of a , $\sqrt{a}$,
ii. Square root of c, $\sqrt{c}$.
iii. Product of 2 , $\sqrt{a}$, $\sqrt{c}$. , ie 2 x $\sqrt{a}$ x$\sqrt{c}$.

If 2 x $\sqrt{a}$ x$\sqrt{c}$ = b then the quadratic is a Perfect Square.

In our example: $\mathbf{4x^2 + 12x + 9}$; a = 4, b = 12 , c = 9

$\sqrt{a} = \sqrt{4} = 2$, $\sqrt{c} = \sqrt{9} = 3$

2 x $\sqrt{a}$ x$\sqrt{c}$. = 2 x 2 x 3 = 12

Since 2 x $\sqrt{a}$ x$\sqrt{c}$. = 12 = b , then $\mathbf{4x^2 + 12x + 9}$ is a **perfect square.**

To Factorise $\mathbf{4x^2 + 12x + 9} = (\sqrt{(ax^2)} + \sqrt{c})^2$

$= (\sqrt{(4x^2)} + \sqrt{9})^2$

$= (2x+3)^2$

More examples:

Factorise	**Check**:
Eg. 1 $4x^2 + 20x + 25$ Is it a perfect square? Yes, since 2 x $\sqrt{a}$ x$\sqrt{c}$.= b $\therefore$ $4x^2 + 20x + 25$ $= (\sqrt{(4x^2)} + \sqrt{25})^2$ $= (2x+5)^2$	a = 4, b = 20 , c = 25 • $\sqrt{a} = \sqrt{4} = 2$ • $\sqrt{c} = \sqrt{25} = 5$ 2 x $\sqrt{a}$ x$\sqrt{c}$. = 2x2x5=20
Eg. 2 $9x^2 - 24x + 16$ Is it a perfect square? Yes, since 2 x $\sqrt{a}$ x$\sqrt{c}$.= b $\therefore$ $9x^2 + 24x + 16$ $= (\sqrt{(9x^2)} - \sqrt{16})^2$ $= (3x-4)^2$	**Check**: a = 9, b = 24, c = 16 • $\sqrt{a} = \sqrt{9} = 3$ • $\sqrt{c} = \sqrt{16} = 4$ 2 x $\sqrt{a}$ x$\sqrt{c}$. = 2x3x4=24

Eg. 3 $16x^2 - 8x + 1$ Is it a perfect square? Yes, since 2 x $\sqrt{a}$ x$\sqrt{c}$.= b $\therefore$ $16x^2 - 8x + 1$ = $(\sqrt{(16x^2)} - \sqrt{1})^2$ = $(4x - 1)^2$	**Check**: a = 16, b = 8, c = 1 • $\sqrt{a} = \sqrt{16} = 4$ • $\sqrt{c} = \sqrt{1} = 1$ 2 x $\sqrt{a}$ x$\sqrt{c}$. = 2x4x1= 8

Self-Check Exercise 2.2:

Factorise the following :

1. $4x^2 - 28x + 49$
2. $9x^2 - 48x + 64$
3. $4x^2 + 4x + 1$
4. $16x^2 + 88x + 121$
5. $4x^2 - 12xy + 9y^2$
6. $9x^2 + 24xy + 16y^2$

2. *Difference of TWO Squares*

Manatu'i e taimi naa tau expand ai e factor 'e 2'oma'u ai e difference of two squares:

Taimi ia ko e ongo factor 'oku tatau pe ko e kehekehe pe ongo signs.

Eg. (x + 3)(x – 3)
Taimi ko e 'oku expand ai : (a – b)(a +b) = $a^2 - b^2$. This is a difference between two squares.

$$(x - 3)(x + 3) = x^2 - 3^2 = x^2 - 9$$

Now we are doing the factorization, as mentioned earlier it is the opposite process of expanding. That is writing a sum as a product.

Eg. Factorise $x^2 - 9$ both of the terms are squares

ie. x^2 and 9. 9 can be written as 3^2

Since $x^2 - 9$ is a difference between two squares. Which is the product of two factors where corresponding terms are equal where the difference is the sign.

Eg.
$x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$

More examples of Difference between Two Squares

Factorise	
Eg. 1 $100 - x^2$ Is it a Difference between two squares? Yes, $100 - x^2$ is a	**Check**: 100 is a square and can be written as 10^2 x^2 is a square $\therefore$ $100 - x^2$ $= (10)^2 - x^2$ $= (10 - x)(10 + x)$
Eg. 2 $9x^2 - 16$ Is it a Difference between two squares? Yes,	**Check**: $9x^2$ is a square and can be written as $(3x)^2$ 16 is a square and can be written as 4^2 $\therefore 9x^2 - 16 = (3x)^2 - 4^2$ $(3x - 4)(3x + 4)$
Eg. 3 $x^2 - 49$ Is it a Difference between two squares? Yes,	**Check**: x^2 is a square 49 is a square and can be written as 7^2 $\therefore x^2 - 49 = x^2 - 7^2$ $(x - 7)(x + 7)$
Eg. 4 $16x^4 - 81y^4$ Is it a Difference between two squares? Yes,	$16x^4$ is a square and can be written as $(4x^2)^2$ $81y^4$ is a square and can be written as $(9y^2)^2$ $\therefore$ $16x^4 - 81^4$ = $(4x^2)^2 - (9y^2)^2$ $(4x^2 - 9y^2)(4x^2 + 9y^2)$

Self-Check Exercise 2.3:

Factorise these expressions:

1. $x^2 - 36$
2. $9a^2 - 16b^2$
3. $64x^2 - 36$
4. $121x^4 - 4y^4$
5. $81 - x^2$
6. $4x^2 - 1$
7. $49 - 25x^2$

Two Stage Factorising

There are cases where there is no common factor among all terms. We need to pair them first before Factorising.

Eg. 1. Factorise pr + qr + ps + qs

With this eg. there is no common factor for all of the four terms.
To factorize, we need to pair them first. Into two groups.

Eg. can pair pr + qr + ps + qs

Factorise each pair:

pr + qr + ps + qs

r(p+q) + s(p + q)

The factor in green are common then we have

(p + q)(r + s)

Or another pair can be

pr + ps + qr + qs

p(r + s) + q(r + s)

The factor in blue are common then we have

(r + s)(p + q)

More examples

Factorise Eg. 1 xy – xz + wy – wz	Pair. xy – xz + wy – wz x(y – z) + w(y – z) (y – z)(x + w) *OR* xy +wy – xz – wz y(x + w) – z(x +w) (x +w)(y – z)
Eg. 2. a + b – bc – ac	Pair a – ac +b – bc a(1 – c) + b(1 – c) (1 – c)(a +b) ***OR*** a + b – bc – ac (a + b) – c(b +a) (a +b)(1 – c)
Eg. 3 8c – 4d + 2cx – dx	Pair 8c – 4d + 2cx – dx 4(2c – d) + x(2c – d) (2c – d)(4 + x) OR 8c + 2cx – 4d – dx 2c(4 + x) – d(4 + x) (4 + x)(2c – d)

Self-Check Exercise 2.4:

Factorize these expressions:

1. ab + ac + bd + cd
2. xy + xz + wy + wz
3. ef + dh + eh + df
4. st – su + rt – ru
5. ce + cf – 2de – 2df
6. 3pq – 3pr – 2ps + 2rs
7. 6fg – 28eh + 8gh – 21ef
8. 2ax + 2y – ay – 4x
9. 3xy + 6ay – 4az – 2xz
10. (4x – 3y)(a + b) – 4x + 3y

Factorizing :

Quadratics when in the form $ax^2 + bx + c$

1. when a = 1 then we have $\boldsymbol{x^2 + bx + c}$

- look for two factors of c where their sum is b.

$x^2 + bx + c = x^2 + (e+f)x + ef$

where e + f = b and ef = c.

Eg. 1. Factorize $x^2 + 7x + 12$

Factors of 10 ; in pairs

1, 12 ; 2, 6 ; 3, 4

3 and 4 are factors of 12 where their sum is 7.

∴ To factorize $x^2 + 7x + 12 = (x + 3)(x + 4)$

More examples:

Factorize 2. $x^2 + 11x + 18$ $x^2 + (2+9)x + (2x9)$ $(x + 2)(x + 9)$	Pair of factors of 18: 1,18 ; 2, 9 ; 3, 6 2 + 9 = 11 & 2 x 9 = 18 $x^2 + 11x + 18$ can be written as:
3. $x^2 + 2x - 24$ When c is negative (– 24) One of the two factors is negative. Since b is positive the lowest factor must be negative	Pair of factors of 24: 1,24 ; 2, 12 ; 3, 8 ; 4,6 4 is the lowest factor and should be – 4 – 4 + 6 = 2 & – 4 x 6 = - 24 ∴ $x^2 + 2x - 24$ can be written as: $x^2 + (6+(-4))x + (6x\text{-}4)$ $= (x - 4)(x + 6)$
4. $x^2 + 3x - 40$	Pair of factors of 40: 1,40 ; 2, 20 ; 4, 10 ; 5,8 5 is the lowest factor and should be – 5 ∴ – 5 + 8 = 3 & – 5 x 8 = – 40 ∴ $x^2 + 3x - 40$ can be written as: $x^2 + (8+(-5))x + (8 \text{ x } -5)$ $= (x - 5)(x + 8)$
5. $x^2 - 4x - 21$ When c is negative (– 21) One of the two factors is negative. Since b is negative (– 4) the highest factor must be negative	Pair of factors of 21: 1,21 ; 3, 7 ; 7 is the highest factor and should be – 7 – 7 + 3 = - 4 & – 7 x 3 = - 21 ∴ $x^2 + 2x - 24$ can be written as: $x^2 + (3+(-7))x + (3x\text{-}7)$ $= (x - 7)(x + 3)$
6. $x^2 - 7x - 30$ When c is negative (– 30) One of the two factors is negative. Since b is negative (– 7) the highest factor must be negative	Pair of factors of 21: 1,30 ; 2,15 ; 3,10 ; 5,6 10 is the highest factor and should be – 10 – 10 + 3 = - 7 & – 10 x 3 = - 30 ∴ $x^2 - 7x - 30$ can be written as: $x^2 + (3+(-10))x + (3 \text{ x-}10)$ $= (x - 10)(x + 3)$
7. $x^2 - 5x + 6$ When c is positive (+6) And b is negative (– 5) both factors are negative and their sum is b.	Pair of factors of 6: 1,6 ; 2,3 Both factors must be negative - 2 and – 3 – 2 + -3 = - 5 & – 2 x -3 = 6 ∴ $x^2 - 5x + 6$ can be written as: $x^2 + (-3+-2)x + (-3 \text{ x-}2)$ $= (x - 3)(x - 2)$
8. $x^2 - 9x + 20$ When c is positive (+20) And b is negative (– 9) both factors are negative and their sum is b.	Pair of factors of 6: 1,20 ; 2,10 ; 4,5 Both factors must be negative - 4 and – 5 – 4 + - 5 = - 9 & – 4 x -5 = 20 ∴ $x^2 - 9x + 20$ can be written as:$x^2 + (-5+-4)x + (-5 \text{ x-}4)$ $= (x - 5)(x - 4)$

Self-Check Exercise 2.5

Factorize these expressions:

1.	$x^2 + 11x + 30$	7.	$x^2 + 8x + 7$
2.	$x^2 + 16x + 28$	8.	$x^2 + 2x - 120$
3.	$x^2 + 5x - 6$	9.	$x^2 + 11x - 60$
4.	$x^2 - 8x - 20$	10.	$x^2 - 3x - 40$
5.	$x^2 - x - 56$	11.	$x^2 - 11x + 24$
6.	$x^2 - 12x + 32$	12.	$x^2 - 14x + 45$

2. when $a > 1$ then we have $ax^2 + bx + c$

Case A : Where there is a common factor for all terms

- pull out the common factor and you will have a format like when a = 1

$$x^2 + bx + c$$

Examples

Factorize : 1. $2x^2 - 10x - 12$	• Pull out the common factor $2(x^2 - 5x - 6)$ $2(x - 3)(x + 2)$
2. $4x^2 + 16x - 48$	• Pull out the common factor $4(x^2 + 4x - 12)$ $4(x - 2)(x + 6)$
3. $5x^2 + 55x + 140$	• Pull out the common factor $5(x^2 + 11x + 28)$ $2(x + 4)(x + 7)$

Self-Check Exercise 2.6

Factorise therse expressions

1. $2x^2 - 2x - 12$
2. $4x^2 + 20x + 16$
3. $3x^2 + 3x - 90$
4. $6x^2 - 84x - 192$

Case B : Where there is NO common factor for all terms.

Eg. $4x^2 + 11x + 6$

- You can use trial and error in this case by examine all the pair of factors .

  Eg. possible pairs of factors which give $4x^2$ and 6 are :

  (4x + 6)(x+1), (4x+1)(x + 6), (4x+3)(x+2)
  (4x +2)(x+3), (2x+6)(2x+1),(2x+2)(2x+3)

  (4x+3)(x+2) is the only pair that give $4x^2 + 11x + 6$ when expanded

However, you can do another approach to expand this case. By using the Two Stage Factorising.

Steps to follow: $ax^2 + bx + c$

1. Multiply a and c : ac
2. Find two factors of ac where their sum is b.
3. Call this number p and q
4. Write $ax^2 + bx + c$ **as** $ax^2 + px + qx + c$
5. Then factorise using Two Stage Factorizing.

Examples

Factorize : 1. $4x^2 + 11x + 6$ 4 x 6 = 24 Two factors of 24 added to 11 are 3 and 8.	Rewrite $4x^2 + 11x + 6$ as $4x^2 + 8x + 3x + 6$ Pair: $4x^2 + 8x + 3x + 6$ $4x(x + 2) + 3(x + 2)$ Pull out common factor $(x + 2)(4x + 3)$
2. $6x^2 + 5x + 1$ 6 x 1 = 6 Two factors of 6 added to 5 are 3 and 2.	Rewrite $6x^2 + 5x + 1$ as $6x^2 + 3x + 2x + 1$ Pair: $6x^2 + 3x + 2x + 1$ $3x(2x + 1) + 1(2x + 1)$ Pull out common factor $(2x + 1)(3x + 1)$

3. $12x^2 + x - 6$ $12 \times -6 = -72$ Two factors of -72 added to 1 are -8 and 9.	Rewrite $12x^2 + x - 6$ as $12x^2 - 9x + 8x - 6$ Pair: $12x^2 - 9x + 8x - 6$ $3x(4x - 3) + 2(4x - 3)$ Pull out common factor $(4x - 3)(3x + 2)$
4. $3x^2 - 10x - 8$ $3 \times -8 = -24$ Two factors of -24 added to -10 are -12 and 2	Rewrite $3x^2 - 10x - 8$ as $3x^2 - 12x + 2x - 8$ Pair: $3x^2 - 12x + 2x - 8$ $3x(x - 4) + 2(x - 4)$ Pull out common factor $(x - 4)(3x + 2)$
5. $5x^2 - 18x + 9$ $5 \times 9 = 45$ Two factors of 45 added to -18 are -3 and -15	Rewrite $5x^2 - 18x + 9$ as $5x^2 - 15x - 3x + 9$ Pair: $5x^2 - 15x - 3x + 9$ $5x(x - 3) - 3(x - 3)$ Pull out common factor $(x - 3)(5x - 2)$

.

Self-Check Exercise 2.7

Factorise these expressions:

1. $2x^2 + 7x + 3$
2. $3x^2 - 7x + 2$
3. $2x^2 - 13x - 24$
4. $3x^2 + 20x - 7$
5. $12 + 5x - 2x^2$
6. $2x^2 + 11x + 12$
7. $7x^2 - 22x + 3$
8. $6x^2 - x - 12$
9. $5x^2 + 19x - 4$

Activity Sheet 2.1

Find attached