Fanau: Malo 'etau ma'u e 'aho ni.

Kataki Ko e veesi folofola ena ke kamata 'aki 'etau Kalasi. Lau ia, pea fai ha'o lotu pea toki kamata ho'o ngaahi ngaue:

Paloveape 3: 5 &6

" *Falala ki he 'EIKI 'aki 'a e kotoa ho loto 'o 'oua 'e faaki ki he poto 'o'ou. Ke ke fakaongo kiate ia 'i ho hala kotoa pe, pea 'e fakatonutonu 'e ia ho ngaahi 'alunga* "

STRAND : ALGEBRA **(Notes #3)**

Solving of Quadratics:

Quadratics Equation(Q.Eq): Any equation that can be written in $ax^2+bx + c = 0$ where $a \neq 0$

Three Methods for Solving

1. Factorization
2. Completing the Square
3. Quadratic Formula

1. Factorization

Steps to follow:

i. Put all terms on one side of the equal sign, leaving zero on the other side.
ii. Factorizing.
iii. Set each factor equal to zero.
iv. Solve each of these equations.
v. Check by inserting your answer in the original equation.

Eg.1 Solve $x^2 + 2x = 0$

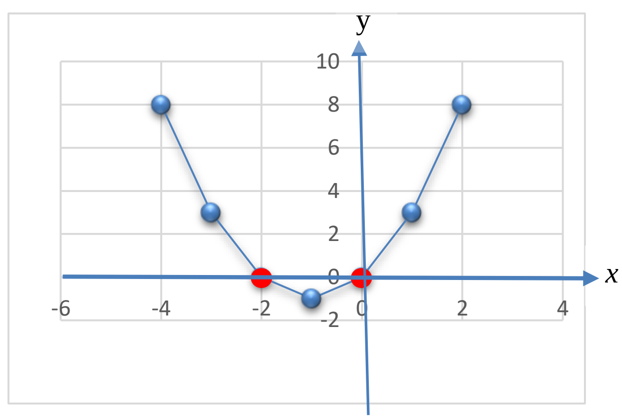
ie. Find the value(s) of x that will make $x^2 + 2x$ equal zero

Steps to follow:	**Activity**
i.	(step i. is complete since all terms are at the left and zero at the right of the equal sign)
ii. Factorize	Write $x^2 + 2x$ as a product of two factors. $x^2 + 2x = 0$ $x(x + 2) = 0$
iii. Set each factor = 0	*For $x(x + 2)$ to be 0 one or both factors must be zero* ie. If $x = 0$ then $0 * (x + 2) = 0$ OR If $(x + 2) = 0$ then $x * 0 = 0$ $\therefore$ set each factor equal to zero ie. $x = 0$ OR $x + 2 = 0$
iv. Solve each of the equations in iii.	$x = 0$ OR $x + 2 = 0$ $x = 0 - 2$ $\therefore x = 0$ OR $x = -2$
v. Checking *Substitute each value of x into $x^2 + 2x$*	*When $x = 0$, then* $x^2 + 2x = 0^2 + 2(0) = \boldsymbol{0}$ *When $x = -2$, then* $x^2 + 2x = (-2)^2 + 2(-2)$ $= 4 + (-4) = \boldsymbol{0}$

NB. What is the meaning of the answers ie. $x = 0$ and $x = -2$

Geometrically, these are the x-intercepts of the graph of $x^2 + 2x$ or in other words, these are the points that the graph of $x^2 + 2x$ cut (pass through) the x- axis.

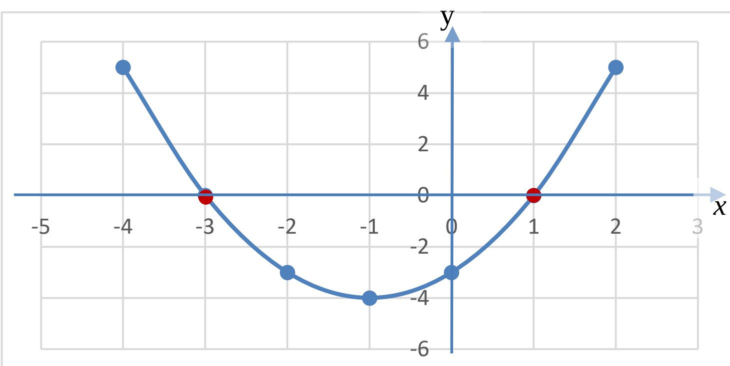
The graph shown below is that of y = $x^2 + 2x$.
The x – intercepts are highlighted in Red. (ie. when $x = 0$ OR $x = -2$



Eg. 2. Solve $x^2 = -2x + 3$

i. Re-arrange	$x^2 + 2x - 3 = 0$
ii. Factorize	Write $x^2 + 2x - 3$ as a product of two factors. $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$
iii. Set each factor to zero	*For* $(x + 3)(x - 1)$ to be 0 one or both factors must be zero ie. If $(x + 3) = 0$ then $0 * (x - 1) = 0$ OR If $(x - 1) = 0$ then $(x + 3) * 0 = 0$ ∴ set each factor equal to zero ie. $x - 1 = 0$ OR $x + 3 = 0$
iv. Solve each of the equations in iii.	$x - 1 = 0$ OR $x + 3 = 0$ $x = 0 + 1$ OR $x = 0 - 3$ ∴ $x = 1$ OR $x = -3$
v. Checking *Substitute each value of x into* $x^2 + 2x$	*When* $x = 1$, *then* $x^2 + 2x - 3 = 1^2 + 2(1) - 3$ $= 1 + 2 - 3 = 0$ *When* $x = -3$, *then* $x^2 + 2x - 3 = (-3)^2 + 2(-3) - 3$ $= 9 - 6 - 3 = 0$

The graph given below is the graph of $y = x^2 + 2x - 3$
The x-intercepts (points where the graph cut the x-axis-in red) are $x = 1$ or $x = -3$



Eg. 3 Solve $x^2 - 1 = 0$ $x^2 - 1 = 0$ $x^2 = 1$; Take sqrt of both side $x = \pm\sqrt{1}$ $x = 1$ or $x = -1$	$x^2 - 1 = 0$ $(x + 1)(x - 1) = 0$ $(x + 1) = 0$; $(x - 1) = 0$ $x = 0 - 1$; $x = 0 + 1$ $x = -1$; $x = 1$ *You can solve this Q. Eq by Square rooting:*

Eg. 4. Solve $2x^2 + 3x = 2$ $2x^2 + 3x - 2 = 0$ Factorise $2x^2 + 4x - x - 2$ $2x(x + 2) - (x + 2)$ $(x + 2)(2x - 1)$	$2x^2 + 3x - 2 = 0$ $(x + 2)(2x - 1) = 0$ $(x + 2) = 0$, $(2x - 1) = 0$ $x = 0 - 2$, $2x = 0 + 1$ $x = -2$, $2x = 1$ $x = \frac{1}{2}$ ∴ $x = -2$ or $x = \frac{1}{2}$

Self – check 3.1

Solve the following quadratic equations by Factorizing.

a. $x(2x - 3)$
b. $(3 - x)(x + 1)$
c. $x^2 - 49 = 0$
d. $2x^2 - 32 = 0$
e. $100 - 4x^2 = 0$
f. $x^2 + 4x - 21 = 0$
g. $x^2 + 7x - 18 = 0$
h. $x^2 - 3x - 40 = 0$
i. $x^2 - x = 70$
j. $x^2 - 4x + 3 = 0$
k. $x^2 = 11x - 30$
l. $6x^2 + 7x = 3$
m. $3x^2 + 5x + 2 = 0$
n. $2x^2 + 6x + 4 = 0$

2. Completing the Square

History of Completing the Square;
One of the famous problems which introduces the 'completing of the square was;

"What must be the square which, when increased by ten of its own roots, amounts to 39?"

The equation can actually be described in modern-day symbols like this;

$$x^2 + 10x = 39$$

It is solved by adding 25, or 5^2 to both sides of the equation (to get 5, is half of 10). Doing this, the left-hand side of the equation becomes a perfect square.
So we get

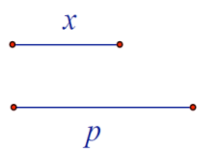
$$x^2 + 10x + 25 = 39 + 25$$
$$(x + 5)^2 = 64$$

This can be easily solved by taking square root of both sides

$$x + 5 = \pm\sqrt{64}$$
$$x + 5 = \pm 8$$

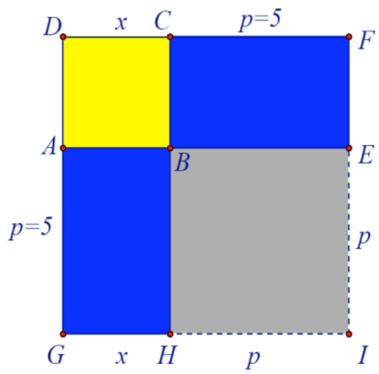
ie. $x + 5 = 8$ or $x + 5 = -8$
$x = 8 - 5$ or $x = -8 - 5$
∴ $x = 3$ or $x = -13$

This is the diagram which represents the problem:



p = 5 cm

Area ABCD = 9 cm²
Area BEFC = 15 cm²
Area AGHB = 15 cm²
Area HIEB = 25 cm²



Can explained this diagram face to face with the class.

More examples:

Eg. 2 Solve $x^2 + 6x - 7 = 0$ Can't factorise $(x+3)^2 = 16$ take square root of both sides $x + 3 = \pm\sqrt{16}$ $x + 3 = \pm 8$ ie. $x + 3 = 8$ 0r $x + 3 = -8$	Let's try Completing the square $x^2 + 6x - 7 = 0$ $x^2 + 6x = 7$; half of 6 = 3 then add 3^2 to both side $x^2 + 6x + 3^2 = 7 + 3^2$ $x^2 + 6x + 9 = 7 + 9$ $(x+3)^2 = 16$ *Now we can solve* $\therefore x + 3 = 8$ or $x + 3 = -8$ $x = 8 - 3$ or $x = -8 - 3$ $x = 5$ or $x = -11$
Eg. 3 Solve $x^2 + 4x = 1$ $(x+2)^2 = 5$ take square root of both sides $x + 2 = \pm\sqrt{5}$ ie. $x + 2 = +\sqrt{5}$ 0r $x + 2 = -\sqrt{5}$	Let's try Completing the square $x^2 + 4x = 1$ $x^2 + 4x = 1$; half of 4 = 2 then add 2^2 to both side $x^2 + 4x + 2^2 = 1 + 2^2$ $x^2 + 4x + 4 = 1 + 4$ $(x+2)^2 = 5$ *Now we can solve* $\therefore x = +\sqrt{5} - 2$ or $x = -\sqrt{5} - 2$ **$x = 0.24$ or $x = -4.24$** (*to 2 decimal places*)

Self-checked # 3.2

Solve the following quadratic equations by "Completing the square"

a. $x^2 + 8x = 33$ d. $x^2 + 10x - 1 = 0$
b. $x^2 + x - 3 = 0$ e. $x^2 + 4x = 21$
c. $x^2 + 6x = 2$ f. $x^2 + 12x = 5$

3. Quadratic Formula

Reminder: Format of quadratic equation is $ax^2 \pm bx \pm c$ where $a \neq 0$.

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

Eg. 1 Solve $x^2 + 6x + 2 = 0$ Quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	Use quadratic formula. From the equation: a = 1 , b = 6 and c = 2 Substitutes these values to the formula; we have $x = \frac{-6 \pm \sqrt{6^2 - 4*1*2}}{2*1}$ $= \frac{-6 \pm \sqrt{36-8}}{2} = \frac{-6 \pm \sqrt{24}}{2}$ ie. $x = \frac{-6 + \sqrt{24}}{2}$ or $x = \frac{-6 - \sqrt{24}}{2}$ $\therefore x = -0.551$ or $x = -5.449$
Eg. 2 Solve $x^2 - 7x + 4 = 0$ Quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	a = 1 , b = –7 , c = 4 Substitutes these values to the formula; we have $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4*1*4}}{2*1}$ $= \frac{7 \pm \sqrt{49-16}}{2} = \frac{7 \pm \sqrt{33}}{2}$ ie. $x = \frac{7 + \sqrt{33}}{2}$ or $x = \frac{7 - \sqrt{33}}{2}$ $\therefore x = 6.372$ or $x = 0.628$

Eg. 3. Solve
$5x^2 = x + 2$

1st Re arrange – we have
$5x^2 - x - 2 = 0$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 5 , b = –1 , c = –2
Substitutes these values to the formula; we have

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-2)}}{2*5}$$

$$= \frac{1 \pm \sqrt{1+40}}{10} \quad = \frac{1 \pm \sqrt{41}}{10}$$

ie.

$$x = \frac{1 + \sqrt{41}}{10} \text{ or } x = \frac{1 - \sqrt{41}}{10}$$

$\therefore\ x = 0.7403$ or $x = -0.5403$

Self – Checked # 3.3

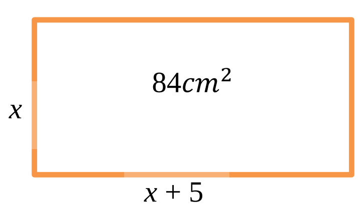
Use the quadratic formula to solve the following quadratic equations.

a. $x^2 + 3x + 1 = 0$
b. $x^2 + 11x + 5 = 0$
c. $x^2 - 6x + 4 = 0$
d. $x^2 - 5x + 2 = 0$
e. $x^2 - 8x - 1 = 0$
f. $7x^2 - x - 4 = 0$
g. $3x^2 + x - 5 = 0$
h. $4 - 9x + x^2 = 0$
I. $7x - 4 = 2x^2$
j. $3x^2 + 4x = 6$

4. Application : Solving of Quadratic equations.

Eg.1 One side of rectangle is 5cm longer than the other. The area is $84cm^2$. Calculate the length of the sides.

Let shorter side be x and longer side be $x + 5$ as shown below.



Area = shorter side * longer side
84 = x * (x + 5)
84 = $x^2 + 5x$
0 = $x^2 + 5x - 84$ or $x^2 + 5x - 84 = 0$

Then factorise: $(x - 7)(x + 12) = 0$

$(x - 7) = 0$ or $(x + 12) = 0$

ie. $x = 0 + 7$ or $x = 0 - 12$

$\therefore\ x = 7$ or $x = -12$

Since we are dealing here with measurement we reject – 12

Our final answer. Lengths of the sides are;
Shorter side is x = 7cm
Longer side is $x + 5 = 7 + 5 = 12$cm

Eg. 2.
The square of two consecutive whole numbers add to 145. What are the two numbers?
If one number is x the next number is x + 1
Given :

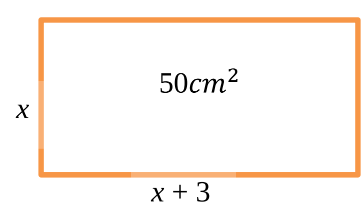
$x^2 + (x+1)^2 = 145$
$x^2 + x^2 + 2x + 1 = 145$
$2x^2 + 2x + 1 = 145$
$2x^2 + 2x + 1 - 145 = 0$
$2x^2 + 2x - 144 = 0$ (Next factorise)
$2(x^2 + x - 72) = 0$
$x^2 + x - 72 = \frac{0}{2}$
$(x + 9)(x - 8) = 0$
$(x + 9) = 0$ *or* $(x - 8) = 0$
$x = 0 - 9$ or $x = 0 + 8$

$\therefore\ x = -9$ or $x = 8$

We do reject – 9 because it is not a whole number.

So, $x = 8$ and $x + 1 = 9$ ie. the two numbers are 8 and 9.

Eg. 3 One side of a rectangle is 3cm longer than the other. If the area is $50cm^2$, calculate the length of the shorter side to 2 decimal places.



Area = shorter side * longer side
50 = $x * (x + 3)$

$50 = x^2 + 3x$
$0 = x^2 + 3x - 50$ or $x^2 + 3x - 50 = 0$

Since we can't factorise we can use the quadratic formula.

a = 1, b = 3 and c = - 50

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4*1*(-50)}}{2*1}$$

$$= \frac{-3 \pm \sqrt{9+200}}{2} = \frac{-3 \pm \sqrt{209}}{2}$$

ie.

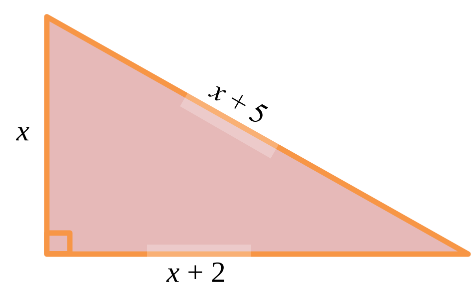
$$x = \frac{-3 + \sqrt{209}}{2} \text{ or } x = \frac{-3 - \sqrt{209}}{2}$$

$\therefore$ $x = 5.728$ or $x = -8.728$

We reject – 8. 728 since we are dealing with length.

$\therefore$ length of shorter side is 5.73cm. (to 2dp)

Eg. 4. The triangle drawn here has sides ; x , x+ 2 and x +5



a. Calculate the value of x correct to 2dp.
Using the Pythagoras Theorem
$c^2 = a^2 + b^2$
$(x + 5)^2 = x^2 + (x + 2)^2$ (expand & Simplify)
$x^2 + 10x + 25 = x^2 + x^2 + 4x + 4$
$x^2 + 10x + 25 = 2x^2 + 4x + 4$
$0 = 2x^2 - x^2 + 4x - 10x + 4 - 25$
$0 = x^2 - 6x - 21$ or $x^2 - 6x - 21 = 0$

We can't factorise $x^2 - 6x - 21 = 0$
Let's use quadratic Formula. :
a = 1 , b = - 6 , c = - 21
Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4*1*(-21)}}{2*1}$$

$$= \frac{6 \pm \sqrt{36+84}}{2} = \frac{6 \pm \sqrt{120}}{2}$$

ie.

$$x = \frac{6 + \sqrt{120}}{2} \text{ or } x = \frac{6 - \sqrt{120}}{2}$$

$\therefore$ $x = 8.48$ or $x = -2.48$

We reject –2.48 since we are dealing with length.

$\therefore$ x = 8.48cm. (to 2dp)

b. Calculate
i. the perimeter of the triangle
Perimeter $= x + (x + 2) + (x + 5)$
= 8. 48 + (8.48 +2) + (8.48 + 5)
= 8. 48 + 10.48 + 13.48
= 32.44cm

ii. the area of the triangle
Area $= \frac{1}{2}$ *base * height
$= \frac{1}{2} * (x + 2) * x$
$= \frac{1}{2} * (10.48) * 8.48$
$= 44.44cm^2$

Self – checked 3.4

Use the most appropriate method to solve these problems:

1. The sides of a rectangle are (x +1) and (x + 9) metres. If the area is $20m^2$, calculate the lengths of the sides.
2. Two consecutive whole numbers multiply to give 156. Find the numbers.
3. A rectangle has a perimeter of 48m and an area of $128m^2$. Calculate its dimensions.

4. One side of a rectangle is 3cm less than the other side. If the area is $117cm^2$, calculate the perimeter of the rectangle.
5. The two shorter sides of a right-angled triangle are $x - 7$ and $x + 2$. If the area of the triangle is $60cm^2$, calculate the length of each of the three sides correct to 4 sf.
6. The perimeter of a rectangular playing field is 170m. If the length of each diagonal of the field is 65m, calculate the dimensions of the field.

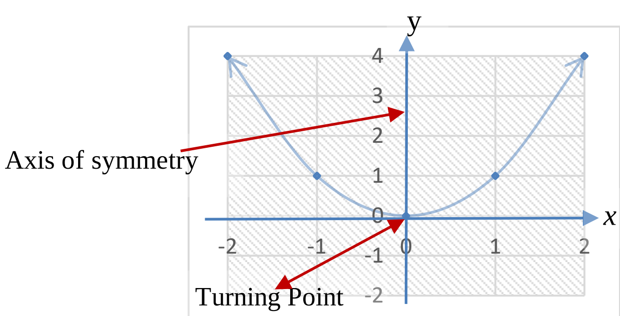
Sketching the graph of quadratics:

The shape for quadratic is known as parabola

Basic features of parabola:

- Shape (even /odd)
- Intercepts (x and y)
- Axis of symmetry
- Turning points
- Domain and range

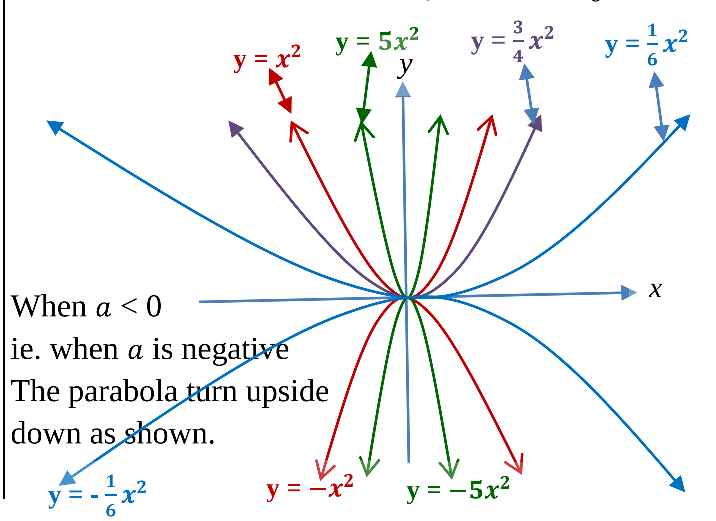
The basic / general equation for quadratic is $y = ax^2$ where $a \neq 0$. Basic Shape when a = 1 : $y = x^2$



Feature of parabola	**Comment**
Shape	Parabola – smooth curve
Intercepts:	x – intercepts – Where the graph cut the x-axis
Axis of symmetry	A line that divide an object into two equal halves (ie. one half is the mirror image of the other half). Therefore the axis of symmetry is the mirror line. Referring to the graph above y-axis is the line that divide the graph into two equal halves therefore y-axis (x = 0) is the axis of symmetry.
Turning point (Vertex)	• It is the point where the graph change from decreasing to increasing (minimum – see our example) or from increasing to decreasing (maximum). The Vertex of the given graph is the origin which is (0, 0).
Domain	• Values of x where the function is defined. Eg. From the graph – the graph is defined for all values of x. Can be written as: Domain = $\{x: x \in R\}$ie. x is an element of Real Numbers.
Range	• Values of y where the function is defined. • Studying the graph for the y-values you have to focus on the graph and the y-axis. • As you can see the graph started from the origin ie. when y = 0 and going up . That shows that the function is only defined when y is greater than or equal to zero. • Can be written as: Range = $\{y, y \geq 0, y \in R\}$ ie. y is greater than or equal to 0, where y is an element of Real Numbers.

Part A : $y = ax^2$

Basic Parabola where the values of a changes. As you can observe, that when $a > 1$ the graph becomes steeper and getting closer to the y-axis like **$y = 5x^2$** (where $a = 5$). However, when $a < 1$ the graph becomes less steep and getting closer to the x-axis like $y = \frac{1}{6}x^2$ (where $= \frac{1}{6}$)



When $a < 0$
ie. when a is negative
The parabola turn upside down as shown.

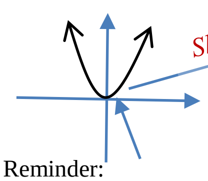
Part B: $y = x^2 \pm C$ where C is a constant

With this format of quadratics – we have to shift the original graph ($y = x^2$) vertically, ie. with +C we shift the graph up and with –C we shift the graph down.

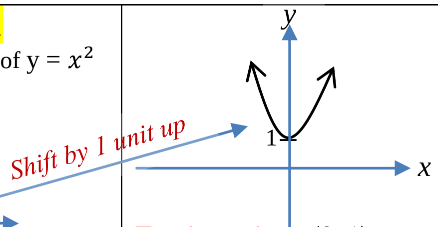
Examples: Sketch the graph of:

1. $y = x^2 + 1$

we shift the graph of $y = x^2$ up by 1 unit.



Reminder:
basic Parabola : $y = x^2$

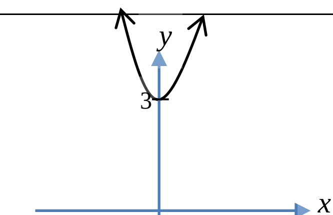


Turning point = (0, 1)
Domain: {x: x ∈ R}
Range: : {y: y ≥ 1, y∈ R}
Axis of symmetry : y-axis (x = 0)

Intercepts. No x-intercepts as you can see that the graph does not pass/cut the x-axis.
y-intercept = 1

2. $y = x^2 + 3$

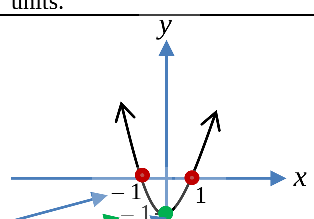
Intercepts. No x-intercepts as you can see that the graph does not pass/cut the x-axis.
y-intercept = 3
Turning point = (0, 3)
Domain: {x: x ∈ R}
Range: : {y: y ≥ 3, y∈ R}
Axis of symmetry: x = 0



we shift the graph of $y = x^2$ up by 3 units.

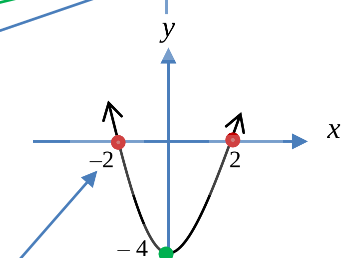
3. $y = x^2 - 1$

we shift the graph of $y = x^2$ down by 1 units.
Intercepts:
x-intercept : when y = 0
ie. $x^2 - 1 = 0$ (solve)
$x = \pm\sqrt{1}$
x = 1 or x = –1
y-intercept = –1
Turning Point = (0, –1)
Domain: {x: x ∈ R}
Range: {y: y ≥ –1, y∈ R}



4. $y = x^2 - 4$

we shift the graph of $y = x^2$ down by 4 units.
Intercepts:
x-intercept : when y = 0
ie. $x^2 - 4 = 0$ (solve)
$x = \pm\sqrt{2}$
x = 2 or x = –2



y-intercept = –4
Turning Point = (0, –4)
Domain: {x: x ∈ R}
Range: {y: y ≥ –4, y∈ R}

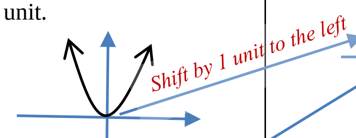
Part C: $y = (x \pm C)^2$ where C is a constant

With this format of quadratics – we have to shift the original graph ($y = x^2$) horizontally , ie. with +C we shift the to the left and with –C we shift the graph to the right.

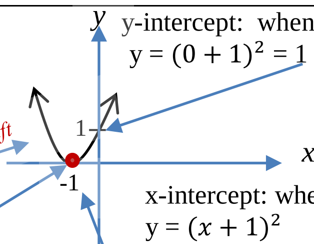
Examples: Sketch the graph of:

1. $y = (x + 1)^2$

we shift the graph of $y = x^2$ to the left by 1 unit.



Turning Point = (-1, 0)
Domain: {x: x ∈ R}
Range: : {y: y ≥ 0, y∈ R}
Axis of symmetry: x = -1



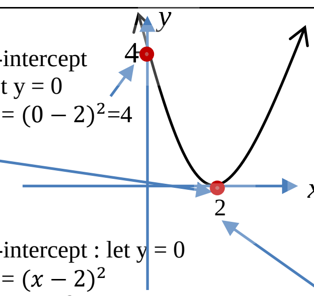
y-intercept: when x = 0
$y = (0 + 1)^2 = 1$

x-intercept: when y=0
$y = (x + 1)^2$
$(x + 1)^2 = 0$:
take sqroot of both side
$x + 1 = 0$
$x = -1$

2. $y = (x - 2)^2$

we shift the graph of $y = x^2$ to the right by 2 unit.
Turning Point = (2 , 0)
Domain: {x: x ∈ R}
Range: : {y: y ≥ 0, y∈ R}
Axis of symmetry: x = 2

y-intercept
let y = 0
$y = (0 - 2)^2 = 4$



x-intercept : let y = 0
$y = (x - 2)^2$
$(x - 2)^2 = 0$; x – 2 = 0 ; x = 2
take sqroot of both side;

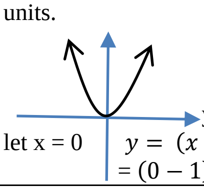
Part C: $y = (x \pm C)^2 \pm k$ where C and k are constants.

With this format of quadratics – we shift the the graph of $y = x^2$ both vertically and horizontally , ie. with +C we shift the to the left and with –C we shift the graph to the right and +k we move up and –k we move down.

Examples: Sketch the graph of:

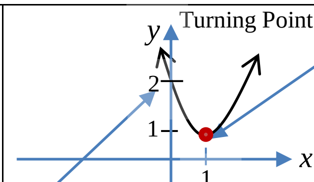
1. $y = (x - 1)^2 + 1$

we shift the graph of $y = x^2$ up by 1 unit and to the right by 2 units.



y-intercept:
let x = 0 $y = (x - 1)^2 + 1$
$= (0 - 1)^2 + 1 = 2$

Turning Point = (1, 1)



The graph does not cross the x-axis ∴ there is no x-intercept

Prepared by Mrs. Losana Vao Latu & Mrs. 'Ana 'Aho

2. $y = (x+1)^2 + 2$

we shift the graph of $y = x^2$ to the left by 1 unit and up by 2 units.

Intercepts:

x-int: the graph does not cross the x-axis ∴ there is no x-int.

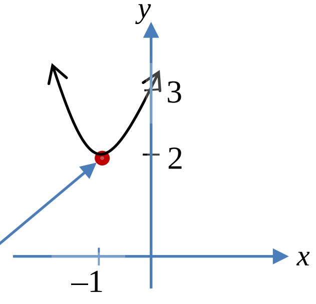
y-int: Let x = 0

$y = (x+1)^2 + 2$

$= (0+1)^2 + 2 = 3$

Turning point: (–1 , 2)

Axis of symmetry: x = – 1



Domain: {x: x ∈ R}

Range: : {y: y ≥ 2, y∈ R}

3. $y = (x-1)^2 - 4$

we shift the graph of $y = x^2$ to the right by 1 unit and down by 4 units.

Intercepts:

x-int: let y = 0

$y = (x-1)^2 - 4$

$(x-1)^2 - 4 = 0$ (solve)

$(x-1)^2 = 0 + 4$

$(x-1)^2 = 4$, take sqroot of both side.

$x - 1 = \pm\sqrt{4}$

$x - 1 = \pm 2$

x – 1 = 2 or x – 1 = – 2

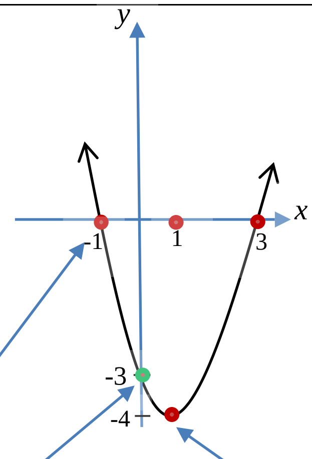
x = 2 + 1 or x = – 2 + 1

x = 3 or x = –1

y-int: Let x = 0

$y = (x-1)^2 - 4$

$= (0-1)^2 - 4 = -3$



Turning point: (1 , - 4)

Axis of symmetry: x = 1

Domain: {x: x ∈ R}

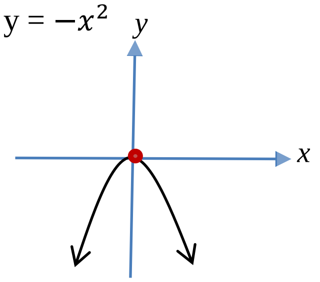
Range: : {y: y ≥ - 4, y∈ R}

Part D: $y = -x^2$

In this part we will consider when the coefficient of the x^2 term is negative and its vertically and horizontally or both translation. ie. $y = -x^2$

Eg. Sketch the graph of ;

$y = -x^2$



x-intercept : x = 0

y-intercept: y = 0

Turning Point : (0, 0)

Domain: {x: x ∈ R}

Range: : {y: y≤ 0 , y∈ R}

Axis of symmetry: y-axis and the equation is x = 0

Vertical Translation

1. $y = -x^2 + 1$

shift the graph of $y = -x^2$ up by 1 unit.

Intercepts:

y-int: Let x = 0

$y = -(0)^2 + 1 = 1$

x-int: Let y = 0

$y = -x^2 + 1$; $-x^2 + 1 = 0$

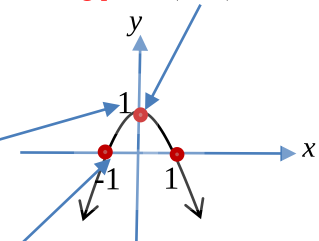
$-x^2 = -1$

$x^2 = 1$; take sqroot of both side

$x = \pm\sqrt{1}$

x = 1 or x = –1

Turning point: (1 , 0)



Axis of symmetry: is the y-axis and the equation is $x = 0$

Domain: {x: x ∈ R}

Range: : {y: y ≤ 1, y∈ R}

2. $y = -x^2 - 2$

shift the graph of $y = -x^2$ down by 2 units.

Axis of symmetry: is the y-axis and the equation is $x = 0$

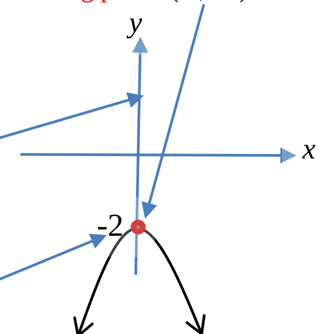
Intercepts

x-int: the graph does not cross the x-axis ∴ there is no x-int

y-int: let x = 0

$y = -0^2 - 2 = -2$

Turning point: (0 , –2)



Domain: {x: x ∈ R}

Range: : {y: y ≤ –2 , y∈ R}

Horizontal Translation

3. $y = -(x-1)^2$

shift the graph of $y = -x^2$ to the right by 1 unit.

Axis of symmetry: is the y-axis and the equation is $x = 1$

Intercepts

x-int: let y = 0

$y = -(x-1)^2$

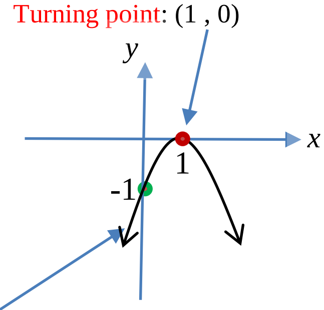
$-(x-1)^2 = 0$; x – 1 = 0

∴ x = 1

y-int: let x = 0

$y = -(0-1)^2 = -1$

Turning point: (1 , 0)



Domain: {x: x ∈ R}

Range: : {y: y ≤ 0 , y∈ R}

4. $y = -(x+2)^2$

shift the graph of $y = -x^2$ to the left by 2 units.

Axis of symmetry: is the y-axis and the equation is $x = 0$

Intercepts

x-int: let y = 0

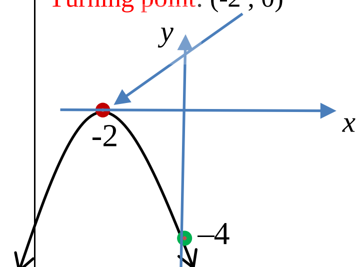
$y = -(x+2)^2$

$-(x+2)^2 = 0$; $x + 2 = 0$

∴ $x = -2$

y-int: let x = 0 $y = -(0+2)^2 = -4$

Turning point: (-2 , 0)



Domain: {x: x ∈ R}

Range: : {y: y ≤ 0 , y∈ R

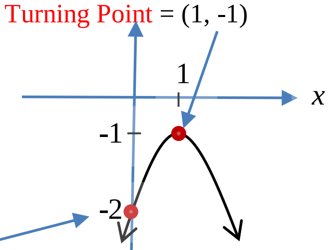
Both vertically & Horizontally Translation

Sketch the graph of:

1. y = $-(x-1)^2-1$
shift the graph of y = $-x^2$ both to the right and down by 1 unit.

Intercepts :
x-int: the graph does not cross the x-axis ∴ no x-int
y-int: let x = 0
y = $-(0-1)^2-1$ = - 2
Axis of Symmetry: x = 1

Turning Point = (1, -1)

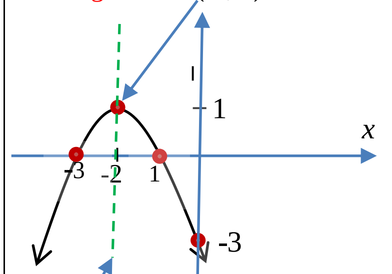


Domain: {x: x ∈ R}
Range: : {y: y ≤ -1 , y∈ R}

2. y = $-(x+2)^2+1$
shift the graph of y = $-x^2$ both to the right and down by 1 unit.

Intercepts :
x-int: let y = 0
y = $-(x+2)^2+1$
$-(x+2)^2+1=0$
$-(x+2)^2$ = –1
$(x+2)^2$ = 1 : sqrt both sides
$x+2=\pm\sqrt{1}$
$x+2$ = 1 , $x+2=-1$
$x = 1-2$, $x=-1-2$
∴ $x=-1$, $x=-3$
y-int: let x = 0
y = $-(0+2)^2+1$ = - 3

Turning Point = (-2, 1)



Domain: {x: x ∈ R}
Range: : {y: y ≤ 1 , y∈ R}
Axis of Symmetry: x = -2

Self-checked: 3.5

Sketch the graphs of the following quadratics and show all the possible features.

1. y = x^2+2
2. y = x^2 - 9
3. y = $-x^2+4$
4. y = $-1-x^2$
5. y = $(x-3)^2$
6. y = - $(x-2)^2$
7. y = $(x+4)^2$
8. y = - $(x+2)^2$
9. y = $(x+2)^2+2$
10. y = $-(x+1)^2-2$
11. y = $-(x-3)^2+1$
12. y = $(x-3)^2+1$
13. y = $-(x-2)^2+4$
14. y = $(x-2)^2-4$
15. y = $(1-x)^2$
16. y = $(x-1)^2-9$

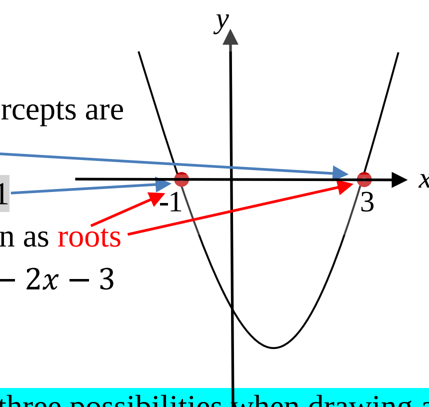
Sketch the graphs of the quadratics in the form of $y = ax^2 \pm bx \pm c$ where $a \neq 0$

Before we continue on with sketching we will go through one special property of Quadratics and this will help a lot when solving and sketching quadratics.

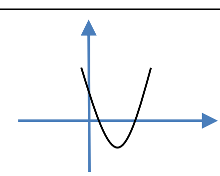
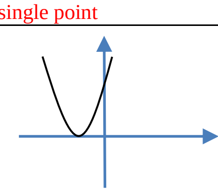
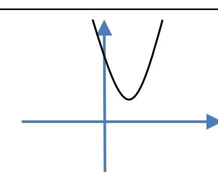
Nature of the **roots** of a quadratic equation:
Ko e ha e roots:
Remember the quadratic equations; $y = ax^2+bx+c$
When you solve; $ax^2+bx+c=0$
The solutions is also known as the roots. Also remember that the solutions is the x-intercepts or the point(s) where the graph cross the x-axis.

Eg. y = x^2-2x-3
Solving $x^2-2x-3=0$ (ie. solving for x when y = 0)
This how you find the x-intercept you let y = 0.
$x^2-2x-3=0$ (solve by factorizing)
$(x-3)(x+1)=0$
$x-3=0$ or $x+1=0$
∴ $x=3$ or $x=-1$ these values of x are known as the roots of y = x^2-2x-3 or known as the x-intercepts or points where y = x^2-2x-3 crosses the x-axis.

The x-intercepts are
x = 3
or x = - 1
also known as roots
of y = x^2-2x-3



There are three possibilities when drawing a parabola.

The parabola may cross the x-axis at two points	The parabola may only touches the x-axis at a single point	The parabola may NOT cross the x-axis at all
 The associated quadratic equation is said to have two real roots	 The associated quadratic equation is said to have one repeated real root	 The associated quadratic equation is said to have No real root

Without drawing the graph you can be able to determine the number of real roots of a quadratic using **discriminant**.

ST.ANDREW'S HIGH SCHOOL
FORM 6 MATHEMATICS- NOTES

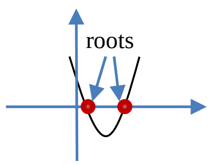
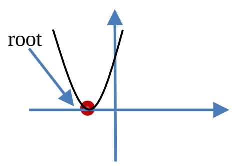
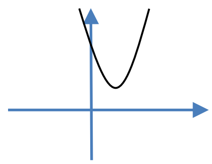
2022

Discriminant: What is it?

If you remember the quadratic formula that we use to solve quadratics equations, $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The discriminant is the number, $b^2 - 4ac$ under the square root sign. It is very important number since it will determine the nature of the real roots of the quadratics.

The parabola may cross the x-axis at two points	The parabola may only touches the x-axis at a single point	The parabola may NOT cross the x-axis at all
 The associated quadratic equation is said to have two real roots	 The associated quadratic equation is said to have one repeated real root	 The associated quadratic equation is said to have No real root
Three cases of the Discriminant:		
$b^2 - 4ac > 0$ The number under the sqrt sign is positive ∴ there are two real roots.	$b^2 - 4ac = 0$ The number under the sqrt sign is zero ∴ one repeated real root, for $x = \frac{-b}{2a}$	$b^2 - 4ac < 0$ The number under the sqrt sign is negative. It is not possible to calculate the square root of a negative number within the real numbers ∴ there are NO real roots.

Examples: Determine the discriminant(D) of the following quadratics and then sketch each graph showing the intercepts and the Turning Point (Vertex).

1. $y = x^2 + 2x - 3$

Discriminant(D): $b^2 - 4ac$

$a = 1, b = 2\ c = -3$

$b^2 - 4ac$

$= 2^2 - 4(1)(-3)$

$= 4 + 12 = 16 > 0$ ie. D is positive ∴ the graph of $y = x^2 + 2x - 3$ has **two real roots** which means the graph crosses the x-axis at two points

Finding intercepts:

y-int: Let x = 0

$y = 0^2 + 2 * 0 - 3 = -3$

x-int: Let y = 0

$y = x^2 + 2x - 3$

$x^2 + 2x - 3 = 0$ (Factorize)

$(x + 3)(x - 1) = 0$

$x + 3 = 0$ or $x - 1 = 0$

$x = 0 - 3$ or $x = 0 + 1$

$x = -3$ or $x = 1$ (roots).

Turning point: (x, y)

(Vertex) = (-1, -4)

Axis of Symmetry: x = -1

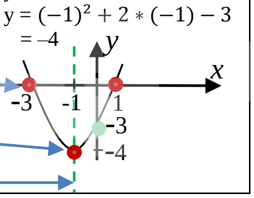
x-coordinate : find midpoint of the roots. ie $x = \frac{-3+1}{2} = \frac{-2}{2} = -1$

y – coordinate: substitute the value of x into

$y = x^2 + 2x - 3$

$y = (-1)^2 + 2 * (-1) - 3$

$= -4$



2. $y = x^2 - 4x - 5$

Discriminant(D): $b^2 - 4ac$

$a = 1, b = -4\ c = -5$

$b^2 - 4ac$

$= (-4)^2 - 4(1)(-5)$

$= 16 + 20 = 36 > 0$ ie. D is +ve

∴ the graph of $y = x^2 - 4x - 5$ has **two real roots** which means the graph crosses the x-axis at two points

Domain: {x: x ∈ R}

Range: : {y: y ≥ -9 , y∈ R}

Finding intercepts:

y-int: Let x = 0

$y = 0^2 - 4 * 0 - 5 = -5$

x-int: Let y = 0

$y = x^2 - 4x - 5$

$x^2 - 4x - 5 = 0$

(Factorize)

$(x + 1)(x - 5) = 0$

$x + 1 = 0$ or $x - 5 = 0$

$x = 0 - 1$ or $x = 0 + 5$

$x = -1$ or $x = 5$ (roots)

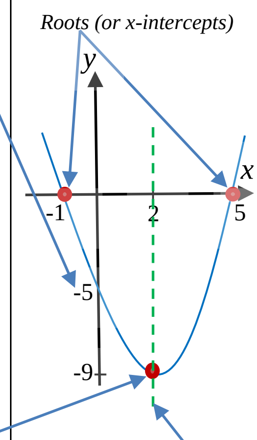
Turning point(TP): (x, y)

x : find midpoint of the roots. ie $x = \frac{-1+5}{2} = \frac{4}{2} = 2$

y : substitute the value of x into $y = x^2 - 4x - 5$

$y = 2^2 - 4 * 2 - 5 = -9$

∴ TP(Vertex) = (2, –9)



3. $y = x^2 - 4x + 4$

Discriminant(D): $b^2 - 4ac$

$a = 1, b = -4\ c = 4$

$b^2 - 4ac$

$= (-4)^2 - 4(1)(4)$

$= 16 - 16 = 0$ ∴ the graph of $y = x^2 - 4x + 4$ has **One repeated real root** which means the graph touches the x-axis at a single point.

Finding intercepts:

y-int: Let x = 0

$y = 0^2 - 4 * 0 + 4 = 4$

x-int: Let y = 0

$y = x^2 - 4x + 4$

$x^2 - 4x + 4 = 0$

(Factorize) If you can notice that $x^2 - 4x + 4$ is a Perfect Square.

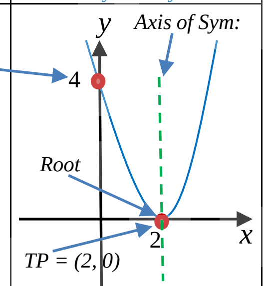
Foki e manatu ki hono factorise of PS.

$(x \pm \frac{b}{2})^2 = (x - \frac{4}{2})^2$

$(x - 2)^2 = 0$ (Sqrt both side)

$x - 2 = 0$

∴ $x = 0 + 2 = 2$



With One repeated real root , the turning point is at the root or the x-intercept as shown above.

4. $y = x^2 + 4x + 4$

Discriminant(D): $b^2 - 4ac$

$a = 1, b = 4\ c = 4$

$b^2 - 4ac$

$= (4)^2 - 4(1)(4)$

$= 16 - 16 = 0$ ∴ the graph of $y = x^2 + 4x + 4$ has **One repeated real root** which means the graph touches the x-axis at a single point.

Finding intercepts:

y-int: Let x = 0

$y = 0^2 + 4 * 0 + 4 = 4$

x-int: Let y = 0

$y = x^2 + 4x + 4$

$x^2 + 4x + 4 = 0$

(Factorize) If you can notice that $x^2 + 4x + 4$ is a Perfect Square.

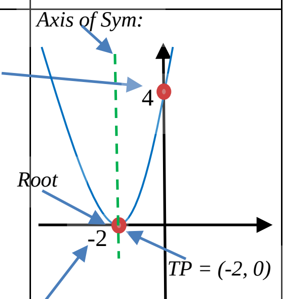
Foki e manatu ki hono factorise of PS.

$(x \pm \frac{b}{2})^2 = (x + \frac{4}{2})^2$

$(x + 2)^2 = 0$ (Sqrt both side)

$x + 2 = 0$

∴ $x = 0 - 2 = -2$



With One repeated real root , the turning point is at the root or the x-intercept as shown above

5. $y = x^2 - 2x + 2$

Discriminant(D): $b^2 - 4ac$

$a = 1, b = -2, \quad c = 2$

$b^2 - 4ac$

$= (-2)^2 - 4(1)(2)$

$= 4 - 8 = -4 < 0$ ie. D is –ve

∴ the graph of $y = x^2 + 4x + 4$ has **No real root** which means the graph does not cross the x-axis.

Finding intercepts:

y-int: Let x = 0

$y = 0^2 - 2 * 0 + 2 = 2$

x-int: Since the graph does not cross the x-axis no need to calculate the x-int. To draw this graph: Write the equation in the form $y = (x \pm c)^2 \pm k$ by completing the square.

Completing the Square:

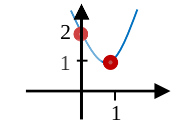
$x^2 - 2x + 2 = 0$

$x^2 - 2x = -2$

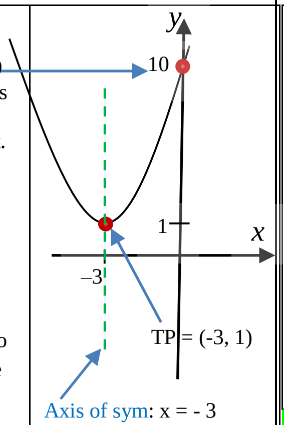
$x^2 - 2x + 1 = -2 + 1$

$(x - 1)^2 = -1$

$(x - 1)^2 + 1 = 0$



Prepared by Mrs. Losana Vao Latu & Mrs. 'Ana 'Aho

6. $y = x^2 + 6x + 10$ Discriminant(D): $b^2 - 4ac$ **a = 1, b = 6, c = 10** $b^2 - 4ac$ $= (6)^2 - 4(1)(10)$ 36– 40 = –4 < 0 ie. D is –ve ∴ the graph of $y = x^2 + 6x + 10$ has **No real root** which means the graph does not cross the x-axis.	Finding intercepts: *y-int*: Let x = 0 $y = 0^2 + 6 * 0 + 10 = 10$ *x-int*: Since the graph does not cross the x-axis no need to calculate the x-int. Complete the square: $x^2 + 6x + 10 = 0$ $x^2 + 6x = 0 - 10$ $x^2 + 6x + 9 = -10 + 9$ $(x + 3)^2 = -1$ $(x + 3)^2 + 1 = 0$ shift the graph of $y = x^2$ to the left 3 units and up one unit. Domain: {x: x ∈ R} Range: : {y: y ≥ 1 , y∈ R}	

2. For what values of p will the roots of $px^2 - 2x + 5 = 0$ be real. To be real : when there is real roots. ie. having either two real roots or one real root. For two real roots. D > 0 For one real root . D = 0 Combining them: D ≥ 0	To have real roots: $b^2 - 4ac \geq 0$ a = p, b = - 2 , c = 5 (substitute) $(-2)^2 - 4 * p * 5 \geq 0$ $4 - 20p \geq 0$ $-20p \geq 0 - 4$ $-20p \geq -4$ (divide by -ve number ∴ reverse the inequality $p \leq \frac{-4}{-20}$ ∴ $p \leq \frac{1}{5}$

Self-checked: 3.6

Sketch the graph of the following quadratics by :

a. first find the value of the discriminant
b. determine the number of real roots
c. calculate the intercepts (x and y)
d. find the Turning point
e. State the Domain and the Range
f. Draw the axis of symmetry and write its equation

1. $y = x^2 - 3x - 4$
2. $y = x^2 - 5x + 6$
3. $y = x^2 + 3x + 2$
4. $y = x^2 - 6x + 9$
5. $y = x^2 + 2x + 1$
6. $y = 2 - x - x^2$
7. $y = -x^2 + 2x + 3$
8. $y = x^2 + 8x + 18$
9. $y = x^2 - 6x + 11$
10. $y = x^2 - x - 12$
11. $y = x^2 + 3x - 10$
12. $y = x^2 + 10x + 25$

Application of Properties of the Discriminant:

Examples:

1. The equation $3x^2 - 4x + q = 0$ has two real roots. Find the range of possible values for q.	To have two real roots the discriminant is greater than zero ie. $b^2 - 4ac > 0$ a = 3, b = - 4 , c = q (substitute) $(-4)^2 - 4 * 3 * q > 0$ 16 – 12q > 0 – 12q > 0 – 16 – 12q > – 16 (divide by -ve number, ∴ reverse the inequality $q < \frac{-16}{-12}$ ∴ $q < \frac{4}{3}$

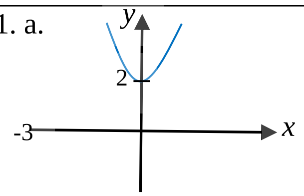
Self – Checked : # 3. 7

1. Find the range of values for q for which the equation $2x^2 - 8x - q = 0$ has two real roots.
2. For what values of a will the roots of $ax^2 - 2x + 5 = 0$ be equal?
3. Find the values of q for which the equation $\frac{4x^2}{5} + 3x - q = 0$ has no real roots.
4. The equation $x^2 - rx + 25 = 0$ has equal roots. Find the two possible values of r.
5. What condition must be placed on p if the roots of the equation $x^2 + 8x + p = 0$ are real and unequal?
6. The equation $4x^2 - 3x + 2q = 0$ has two real roots. Find the range of possible values for q.
7. For what values of b will the roots of $x^2 + bx + 9 = 0$ be real and distinct?
8. Find the value(s) of q such that the equation $4qx = 2x^2 + 7$ has equal roots.
9. For what values of d will the roots of $\frac{1}{4}x^2 + dx - 8 = 0$ be real.
10. The polynomial $(s + 1)x^2 - (s - 2)x + 1$ is a perfect square. Find the value(s) of s.

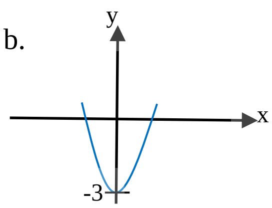
Writing the equation of quadratics given the graph.

Examples: Write the equation of the following parabolas.

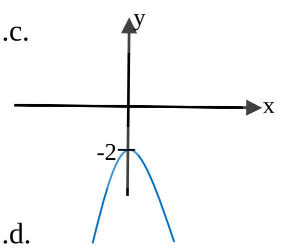
1. a.



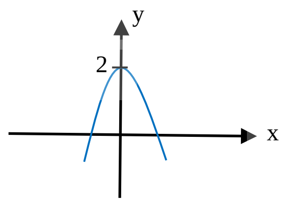
1. b.



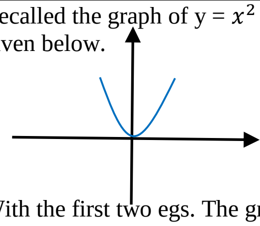
1.c.



1.d.

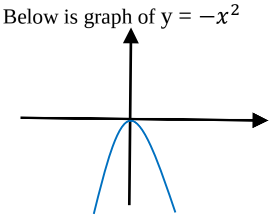


Recalled the graph of y = x^2 as given below.



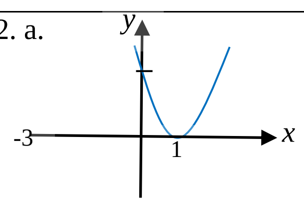
With the first two egs. The graph is being moving up or down with the vertex still on the y-axis. ie. y = x^2 is being **vertically Translated:**
The format of equation is
$y = x^2 \pm C$ where +C when moved up and –C when move down.
∴ Eg. 1. a. y = x^2 is moved up by 2 unit. ie. the equation is
$y = x^2 + 2$
Eg. 1.b. y = x^2 is moved down by 3 units, ie. the equation is
$y = x^2 - 3$

Below is graph of $y = -x^2$

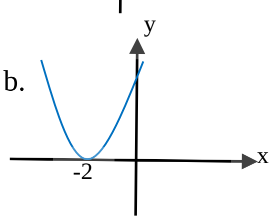


Eg. 1. c. the graph of $y = -x^2$ is move down by 2 units ; the equation is $y = -x^2 - 2$
Eg. 1.d. the graph of $y = -x^2$ is moved up by 2 units: the equation is $y = -x^2 + 2$

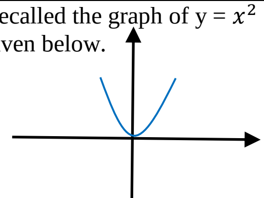
2. a.



2. b.

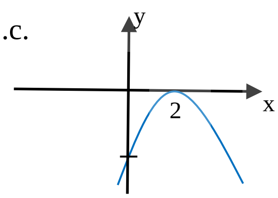


Recalled the graph of y = x^2 as given below.

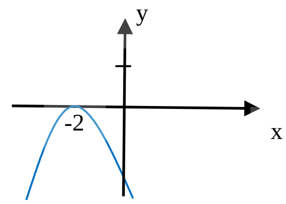


With the first two egs. The graph is being moving left or right with the vertex on the x-axis. ie. y = x^2 is being **Horizontally Translated:**
The format of equation is
$y = (x \pm k)^2$ where +k when moved to the left and – k when move to the right.
∴ Eg. 2. a. y = x^2 is moved to the right by 1 unit. ie. the equation is
$y = (x-1)^2$
Eg. 1.b. y = x^2 is moved to the left by 2 units, ie. the equation is
$y = (x+2)^2$

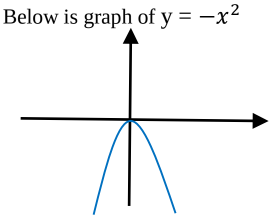
2.c.



2.d.

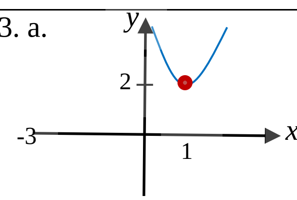


Below is graph of $y = -x^2$

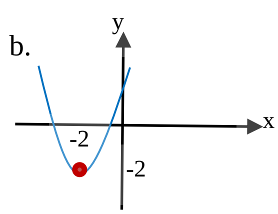


Eg. 2. c. the graph of $y = -x^2$ is move to the right by 2 units ; the equation is $y = -(x-2)^2$
Eg. 2.d. the graph of $y = -x^2$ is moved to the left by 2 units: the equation is $y = -(x+2)^2$

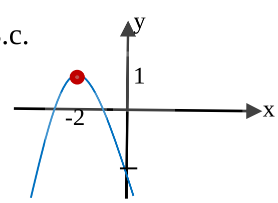
3. a.



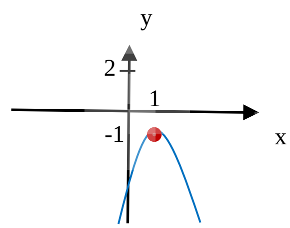
3. b.



3.c.



3.d.



If the vertex(TP) is known, then We can use the format of equation for combining **Vertically and Horizontally Translation**: ie.

$y = (x \pm k)^2 \pm C$

Eg. 3.a. the vertex is (1, 2) ie. the graph of y = x^2 is being moved to the right by 1 unit and twice up.
∴ the equation is $y = (x-1)^2 + 1$

Eg. 3.b. the vertex is (-2 , -2) ie. the graph of y = x^2 is being moved to the left by 2 units and twice down.
∴ the equation is $y = (x-1)^2 + 1$

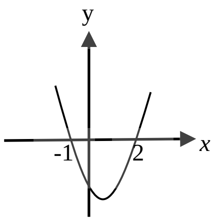
Eg. 3.c. the vertex is (-2 , -2) ie. the graph of $y = -x^2$ is being moved to the left by 2 units and up 1 unit.
∴ the equation is $y = -(x+2)^2 + 1$

Eg. 3.d. the vertex is (1 , -1) ie. the graph of $y = -x^2$ is being moved to the right by 1 unit and down 1 unit.
∴ the equation is $y = -(x-1)^2 - 1$

Let's observe different possibilities of parabolas and how to write their equation.

where the intercepts are known with unknown the TP

a.



Eg. a.

Only the x-intercepts is known. ie. x = -1 or x = 2

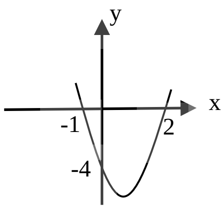
- Rearrange the equations and make one side zero.
  x + 1 = 0 or x – 2 = 0
- Remember when we solve the quadratic equation by factorizing (*we are reversing the process in this case*).

(x + 1)(x – 2) = 0

∴ y = (x + 1)(x – 2) we can expand and have

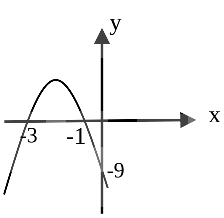
$y = x^2 - x - 2$

b.



Eg. b.
Both the intercepts (x & y) are known
x = -1 or x = 2
x + 1 = 0 or x – 2 = 0
(x + 1)(x – 2) = 0
y – int = - 4 but 1 * - 2 = - 2
∴we need to multiply the equation by 2
2(x + 1)(x – 2) = 0
∴ y = 2(x + 1)(x – 2) we can expand and have
$y = 2(x^2 - x - 2)$
$y = 2x^2 - 2x - 4$

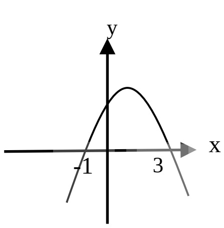
c.



x = - 3 or x = - 1
x + 3 = 0 or x + 1 = 0
(x + 3)(x + 1) = 0
y-int: = - 9 but 3 * 1 = 3
we have to multiply by -3
-3(x + 3)(x + 1) = 0
∴ y = -3(x + 3)(x + 1) we can expand and have
$y = -3(x^2 + 4x + 3)$

$y = -3x^2 - 12x - 9$

d.



x = - 1 or x = 3
x + 1 = 0 or x – 3 = 0
(x + 1)(x – 3) = 0
y – int is unknown

∴ y = – (x + 1)(x – 3) we can expand and have $y = -(x^2 - 2x - 3)$

$y = -x^2 + 2x + 3$

Self-checked : #3. 8

Find the equation for each of the parabola drawn below.