Strand 1: Numbers
Prime numbers, Factors and Multiples, Powers

Learning Outcomes
The prime numbers, common factors and multiples, and powers (including square roots).

- Find lowest common multiples (LCM) of 3 or 4 numbers
- Find highest common factor (HCF) of 3 or 4numbers
- Determine the prime factors of a numbers in index form
- Use calculator to evaluate power number expressions
- Use calculator to find roots of numbers
- Calculate number expressions that involves powers and roots
- Solve problems that involves HCF or LCM
- Multiply and/or Divide power number expression that can be simplifies to the same based
- Interchangeably write number in terms of powers and square roots
- Expanding brackets
- Use the order of operations to calculate numerical expressions (BEDMAS)
- Solve word problems that involves integers and whole numbers

Multiples and lowest common multiple

The **multiples** of a number are obtained by multiplying that number by 1, 2, 3, … in turn. A natural number has an infinite number of multiples.

Example

6 is regarded as a multiple of 6.

Q. Find the first five multiples of 6
A. Multiply the number 6 by 1, 2, 3, 4, 5 in turn:
$6 \times 1 = 6, 6 \times 2 = 12, 6 \times 3 = 18, 6 \times 4 = 24, 6 \times 5 = 30$
The first five multiples of 6 are 6, 12, 18, 24, 30.

The **lowest common multiple** (**LCM**) of two or more numbers is found by listing the multiples of each number and picking out the lowest multiple in common.

Example

Q. Find the LCM of 6 and 8
A. Multiples of $6 = \{6, 1, 18, \mathbf{24}, 30, 36, ... \}$
Multiples of $8 = \{8, 16, 24, 32, 40, 48, ... \}$
24 is the lowest number that is common to both sets.
The LCM of 6 and 8 is 24.
Practical problems can often be solved using multiples.

Example

Q. Tap A drips water every 4 seconds and tap B drips every 5 seconds. If the taps start dripping together, after how many seconds do they next drip together?

A. Find a set of multiples for each tap.

Tap A drips at 4, 8, 12, 16, 20, 24,....seconds
Tap B drips at 5, 10, 15, 20, 25,....seconds

After 20 seconds, the two taps drip together

Activity 1.3

Multiples and LCM

1. **a.** List the multiples of 18 between 100 and 200.
   b. What is the next number in this list of multiples?...,102, 119, 136, 153, …
2. List the next 5 multiples of 7 greater than 100.
3. Find: **a.** the first 5 multiples of 14   **b.** the first 5 multiples of 21
   c. the LCM of 14 and 21
4. Find the LCM of 12 and 18.
5. If x is a multiple of y, find the LCM of x and y.
6. Lina goes training every 6 days and Emma trains every 10 days. If they start training together on Monday, when do they next train together?
7. The lowest common multiple of two numbers is 35. If one of the numbers is 5, find the other number that is less than 35.
8. Mata shops every 6 days and Pina shops every 4 days. If they shop together on a Monday, on what day of the week will they next shop together?
9. A 'leap' year is every 4 years, a 'stem' year is every 5 years and a 'jump' year is every 6 years. If all three types of year occur in the year 2050, when will they have occurred together prior to that year?
10. Two racing tortoises set off together at the start of a circular track. Tina takes 6 hours to complete a lap and Tesi takes 8 hours for a lap. When do they next cross the start line together?

Factors and highest common factor

A natural number that divides exactly into another natural number, without a remainder, is a **factor** of that number.

The number 1 is a factor of all natural numbers. Every natural number is a factor of itself.

8 is a factor of 24, because 8 divides exactly into 24 without a remainder.

5 is not a factor of 24 as 5 divides into 24 four times with a remainder of 4.

A factor that is shared by two numbers is a **common factor**.
The largest common factor is called the **highest common factor** (**HCF**)

Activity 1.4

Factors and HCF
1. List all the factors, in ascending order, of the following numbers.
   a. 78　　　**b.** 108　　　**c.** 91　　　**d.** 200
2. Find the HCF of the following sets of numbers.
   a. 72 and 54　　　**b.** 68, 85 and 102
3. Five of the six factors of a number are {1, 2, 3, 7, 21}. What is the number?
4. Talita, a head teacher, is organizing a trip to Fāʻonelua Park for 120 children. She plans to separate the children into groups with an equal number in each group. She has plenty of teachers and parents to be group supervisors, with one adult allocated to each group. A group will consist of at least 10 children but no more than 20.
   a. How many possible group sizes are there, and how many children in each group?
   b. Find two numbers that are both less than 20, which have exactly 6 factors.

Space for exploration
1. The product of three consecutive counting numbers is always a multiple of 6:
   e.g. $5 \times 6 \times 7 = 210$, which is a multiple of 6 because $210 = 6 \times 35$
   Is this rule always true?
2. The product of four consecutive counting numbers is always divisible by 24:
   e.g. $7 \times 8 \times 8 \times 10 = 5\,040$, which is a multiple of 24 because $24 \times 210 = 5\,040$
   Is this rule always true?
3. The product of five consecutive counting numbers is always divisible by 120:
   e.g. $7 \times 8 \times 9 \times 10 \times 11 = 55\,440$, which is a multiple of 120 because $120 \times 462 = 55440$
   Are there any exceptions?

Divisibility

Certain strategies have been developed to help with the operation of division of numbers.
The following **divisibility tests** can be useful.

Test 1 A number is divisible by 10 if the number ends in 0.

Test 2 A number is divisible by 5 if the number ends in 0 or ends in 5.

Test 3 A number is divisible by 2 if it is even, i.e., ends in 0, or 2, 4, 6, 8.

Test 4 A number is divisible by 3 if the sum of its digits is divisible by 3.

Test 5 A number is divisible by if the number formed by the last two digits is divisible by 4.

Test 6 A number is divisible by 6 if its last digit is even and the sum of its digits is divisible by 3.

Test 7 A number is divisible by 9 if the sum of its digits is divisible by 9.

Activity 1.5

Divisibility

Using the divisibility tests, answer the following questions.

1. Which of the numbers 348, 423, 367, 504, is divisible by 9?
2. Explain why the number 1 5 66 is divisible by both 6 and 9.
3. From the set {10 767, 8 215, 3 172, 9 333}, identify the numbers:
   a. divisible by 5   **b.** divisible by 9   **c.** divisible by 3   **d.** divisible by 4
4. **a.** Explain why the number 6 255 is not divisible by 6.
   b. What is the nearest number less than 6 255 which is a multiple of 6?

Activity 1.6

Primes and prime factors

1. Write down all the prime numbers that:
   a. lie between 10 and 20   **b**. are factors of 36
   c. are multiples of 7   **d.** lie between 100 and 120
2. Express the following numbers as the product of primes, and for each pair state the HCF.
   a.   30 and 105   b.90 and 110
3. a. Find two consecutive prime numbers that lie between 50 and 100 whose difference is 2.
   b. Write down the first nine prime numbers as terms of a sequence. List the difference between the terms. What do you notice?

Powers of Numbers

When a number is multiplies by itself many times, it is simple to express it as a power of the number:

$$\text{e.g.} \quad 81 = 3 \times 3 \times 3 \times 3 = 3^4$$

In words, 81 is equal to 3 to the power 4.

3 is the **base** and 4 is the **power** to which the base is raised.

The *power* can be called the **exponen**t or the **index**.

Plural of index is indices

Powers can be positive or negative numbers.

Activity 1.7

Powers and roots

1. The squats of the numbers 10 to 20 and the cubes of the first 10 numbers are useful as part of numeracy knowledge. Copy, complete, and learn the following table.

Numbers	10	11	12	13	14	15	16	14	18
Squares					196				
Numbers	1	2	3	4	5	6	7	8	9
Cubes						216			

2. Evaluate: **a.** 7^4 **b.** 3^6 **c.** (26^{100})

3. Evaluate: **a.** $\sqrt{400}$ **b.** $\sqrt{4^3}$ **c.** $\sqrt[3]{6^3}$

4. Evaluate: **a.** 1^{-5} **b.** $1 \div 2^4$ **c.** $\sqrt{5^2 + 12^2}$

5. **a.** Evaluate $\sqrt{4^6 \times 8^2}$ and express the answer as a power of 2

6. a. Using the number line, **estimate** between which two whole numbers the value of $\sqrt{14}$ lies?

Order of operations and integers

BEDMAS is a mnemonic for remembering the order of operations when doing calculations

Brackets
Exponents
Division
Multiplication
Addition
Subtraction

> Take care! In some examples, brackets are 'understood' to be in place.

Activity 1.8

Order of operations

1. Evaluate:

    a. $-2 \times -3 + +4$ **b.** $-2 - -2 \times +5$ **c.** $-3 \times +2 + +5 \times -4$

    d. $-3 \times -2 - 5 \times -4$ **e.** $-3 \times -2 + +5 \times +4$ **f.** $5 \times -2 - -6 \times -4$

2. Evaluate:

    a. $\dfrac{-4+2}{3-+1}$ **b.** $\dfrac{-4 \times -3}{-6 + +2}$ **c.** $\dfrac{-4 + -2}{-6 \div +2}$ **d.** $-3-2(3-4)$

    e. $-3 - -2(-3 - +4)$ **f.** $\dfrac{+4 \times -3 - 2}{-2}$

3. Insert a [air of brackets () to make each of the following statements true.

    a. $-3 - -2 \times +5 = -5$ **b.** $-2 \times -5 + 8 = -6$ **c.** $-8 \div +2 - -2 = -2$

4. Insert two pairs of brackets to make this statement true:

    $$-3 + -2 \times +5 \div -4 + -1 = 5$$

5. Find a number exactly halfway between -6 and +2. Write the calculation you have used to get this answer.

Activity 1.9

Problem solving with integers

1. In the Sahara Desert, on a certain day, the temperature was 58 °C. In Antarctica on the same day, the temperature was -46 °C.
   a. How much hotter was it in the Sahara?
   b. The temperature in a hut in Antarctica was 60 °C hotter than the temperature outside. How much hotter it is the Sahara than it was inside the hut in Antarctica?
2. Mt Everest is 8 848 m above sea level, and the Dead Sea is 396 m below sea level. The height of the *Sky City* tower is 328 m above sea level.
   a. What is the difference between the elevations of Mt Everest and the Dead Sea?
   b. What is the difference between the elevations of the Dead Sea and the *Sky City* tower?
3. There are two piles of books in the library – pile A and pile B – that are the same height. Pile A is reduced by 20 cm and B is increased by the same amount. If the height of pile A is now 88 cm, what is now the height of pile B?
4. James keeps track of the level of the lake in the Botanical Gardens. In the middle of the lake, he has erected a post with markers on it every 5 cm. In the middle of the markers is a red line to indicate the optimum depth of the lake as zero. Every Monday James records the mark on the post as above or below the red line and enters the result in a table.

Number of weeks	1	2	3	4	5	6	7	8
Reading in cm	-10	-15	-5	+10	+30	-25	-40	+15

   a. What is the change in the water level from week 1 to week 2?
   b. What is the change in the water level from week 5 to week 6?
   c. From week 8 to week 9 the water level drops by 25 cm. What is James's reading for week 9?
5. Lotu and Naki are playing chess. The winner of a game is awarded 2 points and the loser of a game has 2 points deducted. If a game is drawn, no points are awarded. For the first 8 games, Lotu has the following results: *Lose, win, lose, drawn, win, lose, and win*. What was Naki's points total for the 8 games?