Rational Numbers

Strand 2: Fractions

Learning Outcomes

- Identify with examples of the types of fractions
- Define equivalent fraction
- Write the equivalent representation of a fraction
- Rearrange a set of rational numbers in order
- Add or subtract two fractions
- Simplify fractions
- Multiplying and dividing fractions
- Find fractions when sharing using fractions.
- Find fraction of a fraction
- Find the number when fraction is known
- Solve word problems that involved fraction as part of a whole

Revising fractions

A **fraction** is made up of the ratio of two numbers *a* and *b*, and written as $\frac{a}{b}$

- If b = 0, the fraction is undefined
- a is called the **numerator** of the fraction
- b is called the **denominator**

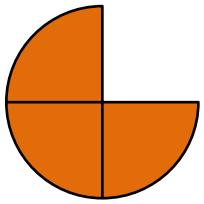
Fractions can be positive or negative.

Three ways to think of a fraction – part of a whole, multiplication, or division.

Part of a whole

Divide a circle into four equal parts.
Take one of the four parts:

- the part taken is $\frac{1}{4}$ of the whole circle
- the part not taken is $\frac{3}{4}$



Multiplication

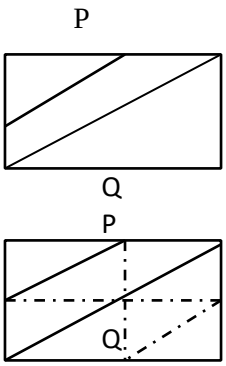
The fraction $\frac{3}{4} = 3$ lots of $\frac{1}{4} = 3 \times \frac{1}{4}$

Division

The fraction $\frac{3}{4} = 3 \div 4$

Fraction are formed when a whole quantity is divided into equal parts.

Example



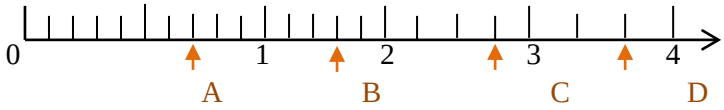
Q. P and Q are the middle points of two sides of a rectangle. What fraction of the rectangle is shaded?

A. One strategy is to divide the rectangle into equal regions, as shown. There are 8 equal triangular regions and 3 are shaded, so $\frac{3}{8}$ of the rectangle is shaded.

Fractions can be represented on a **number line.**

Example

Q. Find the values of the fractions A, B, C, and D, as marked on the number line.



A. In the range 0 to 1 there are 10 divisions. A is at 7 of these divisions, so A is $\frac{7}{10}$

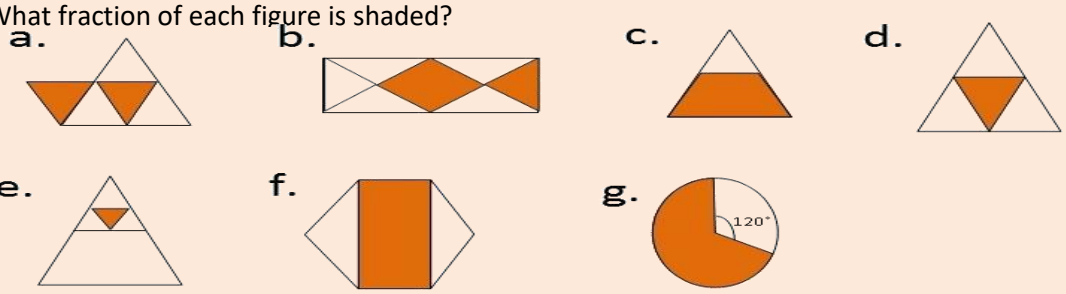
In the range 1 to 2 there are 5 divisions. B is at 3 of these divisions, so B is $1+\frac{3}{5}=1\frac{3}{5}$

Similarly, C is $2\frac{3}{4}$ and D is $3\frac{2}{3}$

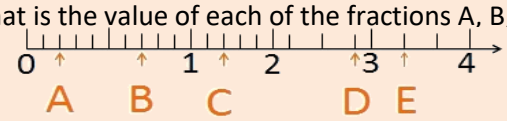
Activity 2.1

Revising fractions

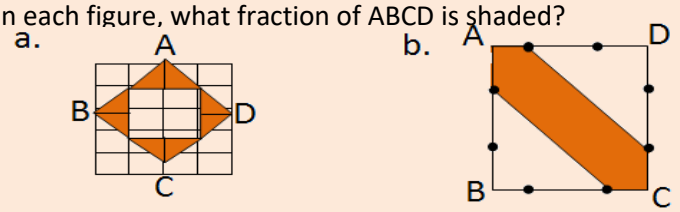
1. What fraction of each figure is shaded?



2. What is the value of each of the fractions A, B, C, D and E?



3. In each figure, what fraction of ABCD is shaded?



Improper fractions and mixed numbers

An **improper fraction** has the numerator larger than the denominator, e.g., $\frac{4}{3}$

A **proper fraction** has the numerator less than the denominator, e.g., $\frac{3}{4}$

A **mixed number** is a fraction added to a whole number, e.g., $1+\frac{1}{3}=1\frac{1}{3}$

Example

Q. Convert $\frac{7}{4}$ to a mixed number.

A. $\frac{7}{4}=\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$ $\qquad \left[\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right]=1$

$=1+\frac{3}{4}=1\frac{3}{4}$

Or $\frac{7}{4}$ = 7 ÷ 4 = 1 remainder 3

The remainder 3 is divided by the denominator 4 to give $\frac{3}{4}$

So, $\frac{7}{4}$ = $1\frac{3}{4}$ again.

Q. Convert $2\frac{1}{3}$ to an improper fraction.

A. $2\frac{1}{3}=2+\frac{1}{3}$

$=\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\frac{1}{3}$

$=7\times\frac{1}{3}$

$=\frac{7}{3}$

Or, $2\frac{1}{3}=\frac{2\times3\times1}{3}=\frac{7}{3}$

Activity 2.2

Improper fractions and mixed numbers

1. Express as mixed numbers:

a. $\frac{15}{4}$ **b.** $\frac{11}{3}$ **c.** $\frac{27}{9}$ **d.** $\frac{45}{13}$ **e.** $\frac{200}{17}$

2. Express as improper fractions:

a. $3\frac{2}{5}$ **b.** $5\frac{6}{7}$ **c.** $14\frac{1}{4}$ **d.** $4\frac{7}{11}$ **e.** $50\frac{3}{50}$

3. **a.** The working for changing an improper fraction into a mixed number is shown:

1 remainder 5

7) 12

What is the improper fraction?

b. The working for changing a mixed number into an improper fraction is shown:

$\frac{5\times7+3}{7}$ What is the mixed number?

Equivalent fractions

Fractions are equivalent when they have the same value.

Suppose a rectangle is divided up in three different ways.

Since equal areas ate shaded in each rectangle, $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$

These three fractions are equivalent, even though they are written in different ways. $\frac{1}{3}$ is the simplest form of these equivalent fractions.

Forming equivalent fractions

To form an equivalent fraction, multiply the numerator and the denominator by the same number. This is the same as multiplying by 1.

Examples

Q. Write five fractions equivalent to $\frac{3}{5}$

A. Multiply $\frac{3}{5}$ by $\frac{1}{1}$, by $\frac{3}{3}$, by $\frac{4}{4}$, and by $\frac{5}{5}$ to obtain five equivalent fractions.

$$\frac{3}{5} \times \frac{1}{1} = \frac{3}{5}$$
$$\frac{3}{5} \times \frac{2}{2} = \frac{6}{10}$$
$$\frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$$
$$\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$$
$$\frac{3}{5} \times \frac{5}{5} = \frac{15}{25}$$

Therefore, five equivalent fractions to $\frac{3}{5}$ are: $\frac{3}{5}$, $\frac{6}{10}$, $\frac{9}{15}$, $\frac{12}{20}$ and $\frac{15}{25}$

Q. $\frac{5}{6} = \frac{?}{42}$ Find the unknown number.

A. This means find the fraction equivalent to $\frac{5}{6}$ that has a denominator of 42.

$\frac{5}{6} = \frac{7}{7} = \frac{35}{42}$ [7 is chosen because $6 \times 7 = 42$]

The unknown number is 35.

Reducing a fraction to its lowest terms

Simplifying a fraction by 'cancelling' common factors is the reverse process to finding equivalent fractions.

Bothe numerator and denominator are divided by the same number.

Example

Q. Express the fraction $\frac{78}{104}$ in its simplest form.

It is important to leave fractions in their lowest terms in an answer.

A. $\frac{78}{104} = \frac{78}{104} \div \frac{2}{2} = \frac{39}{52}$ [both numbers are even, so can be divided by 2]

$\frac{39}{52} = \frac{39}{52} \div \frac{13}{13} = \frac{3}{4}$ [both numbers are multiples of 13]

The fraction $\frac{78}{104} = \frac{3}{4}$ in its lowest terms

Comparing the sizes of fractions

The two fractions $\frac{3}{5}$ and $\frac{4}{5}$ are easy to compare, because they both have the same denominator.

So, $\frac{4}{5} > \frac{3}{5}$ [since 4 > 3]

A strategy to compare fractions with different denominators is to consider a practical example.

Example

Q. Compare the size of $\frac{1}{3}$ with $\frac{1}{4}$

A. Suppose there are 12 paper clips. Take $\frac{1}{3}$ of them. This is 4 paper clips. Take $\frac{1}{4}$ of them. This is 3 paper clips. Since 4 is more than 3, then $\frac{1}{3} > \frac{1}{4}$

Two fractions can also be compared by writing both of them with the same denominator to give equivalent fractions.

Example

Q. Compare the size of $\frac{3}{5}$ with $\frac{4}{7}$

A. $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ and $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$ [using equivalent fractions]

$\frac{21}{35} > \frac{20}{35}$ [the fractions can be compared easily because they have the same denominator]

So, $\frac{3}{5} > \frac{4}{7}$

A variety of fractions can be compared and arranged in order of size by writing all of them with the same denominator.

Activity 2.3

Equivalent fractions

1. For each fraction, write down the next four equivalent fractions.

a. $\frac{3}{4}$ **b.** $\frac{7}{5}$ **c.** $3\frac{4}{11}$

2. Re – write the two fractions $\frac{3}{5}$ and $\frac{7}{10}$ so each has a denominator of 30.

3. Cancel down each fraction to its simplest form:

a. $\frac{9}{30}$ **b.** $\frac{56}{72}$ **c.** $\frac{72}{54}$

4. a. How many fifths in three fifths?

b. How many tenths in three fifths?

5. Arrange these fractions in order of size, smallest first: $\frac{5}{11}, \frac{4}{9}, \frac{3}{7}, \frac{5}{8}, \frac{7}{10}$.

Adding and subtracting fractions

When fractions have the same denominators, they can be added or subtracted easily.

Example

Q. **a.** $\frac{2}{7}+\frac{3}{7}$ **b.** $\frac{7}{8}-\frac{3}{8}$

A. **a.** $\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}$ [denominators the same, add numerators]

$=\frac{5}{7}$

b. $\frac{7}{8}-\frac{3}{8}=\frac{7-3}{8}$ [denominators the same, subtract numerators]

$=\frac{4}{8}=\frac{1}{2}$ [divide numerator and denominator by 4 for answer in simplest form]

Fractions with different denominators have to be re-write with the same denominator, called a **common denominator**, before they can be added or subtracted.

Example

Q. $\frac{5}{6}+\frac{3}{8}$

A. Write a few equivalent fractions to each of the fractions being added.

$\frac{5}{6}=\frac{10}{12}=\frac{15}{18}=\frac{20}{24}=\frac{25}{35}=\cdots$ and $\frac{3}{8}=\frac{6}{16}=\frac{9}{24}=\frac{12}{32}=\cdots$

Select the two fractions that have the same denominator, and then add them together.

$\frac{5}{6}+\frac{3}{8}=\frac{20}{24}+\frac{9}{24}$ [denominators are now the same, so add numerators]

$=\frac{20+9}{24}$

$\frac{29}{24}=1\frac{5}{24}$ [simplify by writing the improper fraction as a mixed number.]

The preceding strategy can be streamlined by identifying the lowest common multiple (LCM) of the denominators 6 and 8.

The LCM of 6 and 8 is 24.

This gives the lowest common denominator (LCD)

Example

Q. $2\frac{3}{10} - 1\frac{7}{15}$

A. The LCM of 10 and 15 is 30; using equivalent fractions, write each fraction with a denominator of 30.

$2\frac{3}{10} - 1\frac{7}{15} = \frac{23}{10} - \frac{22}{15}$ [convert to improper fractions]

$= \frac{23}{10} \times \frac{3}{3} - \frac{22}{15} \times \frac{2}{2}$ [convert to equivalent fractions with denominator 30.]

$= \frac{69}{30} - \frac{44}{30} = \frac{69-44}{30}$ [denominators the same, so subtract numerators]

$= \frac{25}{30}$ [divide numerator and denominator by 5 for answer in simplest form]

$= \frac{5}{6}$

When adding or subtracting mixed numbers, it is often safer to first express mixed numbers as improper fractions. This avoids confusion with negative fractions.

Activity 2.4

Adding and subtracting fractions

Leave each answer in its simplest form.

1. **a.** $\frac{3}{5} + \frac{7}{10}$ **b.** $\frac{2}{3} + \frac{3}{5}$ **c.** $\frac{7}{10} - \frac{3}{5}$ **d.** $\frac{5}{8} - \frac{3}{10}$

2. **a.** $4\frac{1}{3} - 2\frac{3}{5}$ **b.** $2\frac{3}{4} - \frac{7}{10}$ **c.** $\frac{7}{10} + 4$ **d.** $3\frac{7}{30} + 2\frac{3}{8}$

3. The results of an examination indicate that $\frac{2}{5}$ of the students failed. If 99 passed, how many took the exam?

4. Shania spent one third of her money on an ice cream, and two fifths of her money on a drink. If she had $1.20 left, how much did she have to start with?

5. Find two numbers if their sum (i.e. when added) is equal to $\frac{17}{12}$, and their difference (i.e. when subtracted) is equal to $\frac{1}{12}$.

6. a. What number, when subtracted from 2, gives the answer $\frac{5}{6}$?

b. What number, when added to $\frac{3}{4}$, gives the answer $2\frac{5}{8}$?

Multiplying fractions

In the rectangle, $\frac{3}{4}$ is shaded using vertical lines and $\frac{1}{2}$ is shaded using horizontal lines.

The part that is double-shaded is $\frac{1}{2}$ of $\frac{3}{4}$, which is $\frac{1}{2} \times \frac{3}{4}$. The fraction of the rectangle that has both shading is $\frac{3}{8}$. This gives $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

The two numerators have been multiplied together and the two denominators have been multiplied together.

So, $\frac{1}{2} \times \frac{3}{4} = \frac{1\times3}{2\times4} = \frac{3}{8}$

When multiplying mixed numbers, convert them to improper fractions first.

Example

Q. $1\frac{3}{4} \times 1\frac{3}{5}$

A. $1\frac{3}{4} \times 1\frac{3}{5} = \frac{4}{7} \times \frac{8}{5}$ [change to improper fractions]

$= \frac{7\times8}{4\times5}$ [multiply numerators together, multiplying denominators together]

$= \frac{56}{20}$

$= 2\frac{16}{20}$ [change back to mixed numbers]

$= 2\frac{4}{5}$ [write $\frac{16}{20}$ in its simplest form]

Sometimes, cancelling fractions before multiplying will simplify the process.

Remember to change to improper fractions before cancelling.

$1\frac{3}{4} \times 1\frac{1}{5} = \frac{7}{\cancel{4}} \times \frac{\cancel{8}}{5}$ [dividing 4 into 4 and 8]

$= \frac{14}{5} = 2\frac{4}{5}$

Activity 2.5

Multiplying fractions

1. Work out the following:

a. $\frac{2}{3} \times \frac{2}{5}$ **b.** $1\frac{1}{2} \times \frac{1}{3}$ **c.** $\frac{3}{5} \times 2\frac{1}{2}$ **d.** $\frac{5}{8}$ of 12

2. The force of gravity on the surface of the Moon is $\frac{1}{6}$ what it is on Earth.
   a. If a man could jump $2\frac{1}{4}$ meters on Earth, how high could he jump on the Moon?
   b. If a man could jump $9\frac{2}{3}$ meters on the Moon, how high could he jump on the Earth?
3. What number, when multiplied by 2, gives the answer $\frac{1}{6}$?
4. What fraction of $\frac{3}{4}$ is $\frac{3}{8}$?
5. Uncle Etane leaves $52 000 in his will to be shares between his two nephews Logani and Taila. Logani is to inherit $\frac{2}{3}$ of his uncle's money and Taila is to get the rest. Logani decides to give $\frac{1}{10}$ of his money to his favorite charity. How much money does Logani give away?
6. I gave half of my money to my brother, and one third of what was left to my sister, and one quarter of what was then left to my mother. If I had $120 left, how much did I have to start with?

Dividing fractions

One fraction is the **reciprocal** of another fraction if their product is equal to 1:

e.g., if $\frac{2}{3} \times \frac{3}{2} = 1$, then the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

To find the reciprocal of a fraction, turn it upside down.

$5 \div \frac{1}{4}$ is the same thing as saying 'How many quarters are there in 5?'

There are 20 quarters in 5, since $5 \times 4 = 20$

So, $5 \div \frac{1}{4} = 5 \times \frac{4}{1} = 20$ [$\frac{4}{1}$ is the reciprocal of $\frac{1}{4}$]

To divide fractions, multiply the first by the reciprocal of the second fraction.

For example, $\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

Example

Q. $\frac{2}{5} \div 3\frac{1}{4}$

A. $\frac{2}{5} \div 3\frac{1}{4} = \frac{2}{5} \div \frac{13}{4}$ [write $3\frac{1}{4}$ as an improper fraction]

$= \frac{2}{5} \times \frac{14}{13}$ [multiply the first fraction by the reciprocal of the second fraction]

Activity 2.6

Dividing fractions

1. Evaluate: **a.** $\frac{3}{4} \div \frac{2}{3}$ **b.** $\frac{5}{7} \div 2\frac{1}{3}$ **c.** $3\frac{1}{12} \div \frac{12}{13}$

d. $1 \div \frac{3}{5}$ **f.** $3\frac{3}{5} \div 2\frac{1}{4}$

2. Grace uses $3\frac{1}{2}$ kg of sugar to make 21 litres of cordial.

How many litres of cordial would she make from 1 kg of sugar?

3. Following is a list of numbers: 9, 16, 17,18, 24, 48

a. Choose two numbers from the list that one divides by the other to give $\frac{3}{4}$.

b. Choose two numbers from the list that one divides by the other to give $2\frac{2}{3}$.

4. There are two fractions A and B. If A÷B = $\frac{1}{2}$, then:

a. find B, if A = $\frac{2}{3}$ b. find A, if B = $2\frac{4}{5}$

5. What number, when divided by 2, gives the answer $\frac{5}{6}$?

6. What number, when divided by $\frac{5}{8}$, gives the answer $2\frac{5}{8}$?

Problem solving with fractions

Everyday life problems use the four operations of addition, subtraction, multiplication and division of fractions. In some problems, only one operation is needs to get the answer; in other problems, a combination of the operations may be required.

Example

Q. The school PTA raised a total of $30 000 for the year. It was decided to spend $\frac{3}{5}$ of the money on computers. Of this computer money, $\frac{2}{3}$ would be used for laptops.

a. What fraction of the $30 000 would be used for laptops?

b. How much money would be spent on laptops?

A. a. The fraction of the total money spent on laptops is:

$\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{3} \times \frac{3}{5}$

$= \frac{6}{15} = \frac{2}{5}$ [simplifying fraction]

b. The amount of money spent on laptops is:

$\frac{2}{5} \times 30\,000 = \$12\,000$

Activity 2.7

Problems solving with fractions

1. A box of chocolates contains creams, caramels and toffees. After a careful count, it is discovered that $\frac{3}{8}$ of the chocolates are creams, $\frac{1}{4}$ are caramels, and the rest are toffees
   a. What fraction of the chocolates in the box are toffees?
   b. What fraction of the chocolates in the box are either creams or caramels?
   c. If there are 24 chocolates in the box, how many chocolates are caramels?
2. To make pancakes for six people, you need $2\frac{2}{3}$ cups of flour. How many cups of flour would you need to make pancakes for nine people?
3. Amy spent $\frac{3}{8}$ of her money on a drink, and $4.00 left. How much money did she spend?
4. Aloha says her petrol tank is $\frac{1}{4}$ full. When she adds 24 litres of petrol, the tank is then $\frac{7}{8}$ full. How many litres does the tank hold when it is full of petrol?
5. In a class of students, $\frac{5}{6}$ of them like Maths and $\frac{3}{5}$ of them like English. What fraction of the class like both Maths and English?
6. What is $\frac{3}{5}$ of $\frac{3}{5}$ of $100?
7. Sekope is cutting up a length of string. The first length he cuts off is 12 metres. The next length he cuts off is $\frac{3}{4}$ of the precious length. He then cuts off $\frac{3}{4}$ of the precious length again, and so on. After four cuts how much string has, he cut off? Leave your answer as a mixed number.
8. If I divide a certain number by $\frac{1}{3}$, I get the same answer as if I add $\frac{3}{4}$ to the number. What is the number?
9. If one fifth of a certain number is 10, what is three sevenths of the number, expressed as a fraction?

Strand 3: Decimals

Learning Outcomes

- Calculating with decimals
- Adding and Subtracting decimals
- Multiplying and dividing of decimals
- Converting fractions to decimals and decimals to fractions
- Convert decimals to percentage
- Find decimals of amounts expressed as whole numbers, simple fractions and decimals
- Solve number problems that involves addition and subtraction
- Apply addition and subtraction in solving problems in context

Revising decimal place value

A **decimal** or decimal fraction is another way of writing a **fraction.** A decimal:

- is equivalent to a fraction with a denominator of either 10, 100, 1 000, or further powers of 10
- is written using a decimal point to separate whole and fraction parts
- can be positive or negative.

Place value

The first figure after the decimal point represents tenths:

e.g. 0.3 = $\frac{3}{10}$

In words, 0.257 is 'zero point two, five, seven', not 'zero point two hundred and fifty seven'.

Two figures after the decimal point, represent hundredths:

e.g. 0.75 = $\frac{75}{100}$, 0.08 = $\frac{08}{100} = \frac{8}{100}$

Three figures after the decimal point represent thousandths, and so on:

e.g. 0.257 =$\frac{257}{1000}$

If there is no whole numbers, always use a zero before a decimal point, e.g. 0.3

Equals decimals

Zeros can be included at the beginning of the whole number or at the end of a decimal, and make no difference to the value of the total number:

e.g. 2.35 = 2.350 = 2.35000 = 02.35 = 0002.3500, and so on.

Zero at the end of a decimal indicates increased accuracy.

Zero between existing digits will make a difference:

e.g. 2.35 is not equal to 200.35 nor 2.0035, nor 20.000035, and so on.

Activity 3.1

Revising decimals

1. How many hundredths are there in 24 tenths?
2. How many thousandths in 2 tenths?
3. How many hundredths in 235 tenths?
4. What is the place value of the 4 in the following?
   a. 0.0243 **b**. 3.406 **c.** 0.194 **d.** 1.065 914

5. Write in words the following.
   a. 0.26 **c.** 31.07
   b. 6.18 **d.** 1.004 5

Decimals and fractions

Converting fractions to decimals

Fractions that have denominators of 10, 100, 1000, …convert readily to decimals:

e.g. $\frac{7}{10} = 0.7, \frac{31}{100} = 0.31$, and $\frac{29}{1\,000} = \frac{029}{1\,000} = 0.029$

Other fractions can be converted to decimals using different number strategies to change the denominators to multiples of 10. A calculator should not be required.

Example

Q. Convert $\frac{16}{25}$ to a decimal.

A. $\frac{16}{25} = \frac{16}{25} \times \frac{4}{4}$ [convert 25 to 100]

$= \frac{64}{0.64}$

$= 0.64$

If the denominator does not convert easily to a multiple of 10, divide the numerator of the fraction by its denominator. A calculator may need to be used.

Example

Q. Convert $\frac{7}{40}$ to a decimal.

A. The denominator 40 does not convert easily to 10, 100, …

Divide the numerator of the fraction by its denominator.

$7 \div 40 = 0.175$ [using a calculator]

So, $\frac{7}{10} = 0.175$

Converting decimals to fractions
The fraction should be expressed in its simplest form.

Example

Q. Convert 0.64 to a fraction in simplest form.

A. $0.64 = \frac{64}{100} = \frac{16}{25}$ [dividing top and bottom by 4]

Activity3.2

Decimals and fractions

Use mental strategies wherever you can.

1. Write the following decimals as fractions:

a. 0.3 **b.** 0.02 **c.** 3.32 **d.** 0.1212 **e.** 2.085 **f.** 0.0075

2. Write the following fractions as decimals:

a. $\frac{3}{10}$ **b.** $\frac{3}{100}$ **c.** $4\frac{53}{100}$ **d.** $\frac{25}{1\,000}$ **e.** $\frac{10}{10^2}$ **f.** $\frac{304}{100}$

3. Write the following decimals as fractions in their simplest form

a. 0.25 **b.** 0.05 **c.** 0.80 **d.** 2.08

Two kinds of decimals
All fractions when converted into decimals are either **terminating** or **recurring**.

Terminating decimals have a finite number of decimal places:

e.g. $\frac{3}{5} = 0.6$ (1 decimal place), and $\frac{1}{40} = 0.025$ (3 decimal places)

Recurring decimals have an infinite number of decimal places, but repeat a certain pattern of numbers again and again.

$\frac{2}{99} = 0.\mathbf{02}02020202\ldots$ recurs after 2 decimal places.

$\frac{3}{7} = 0.\mathbf{428571}428571428571\ldots$ recurs after 6 decimal places.

To write a recurring decimal, use dots above the *range* of figures that recur (i.e. the first and last figures that recur).

$\frac{2}{3} = 0.\mathbf{6}6666666\ldots$ is written as $0.6\dot{}$

$\frac{2}{99} = 0.\mathbf{02}02020202\ldots$ is written as $0.0\dot{2}$

$\frac{3}{7} = 0.\mathbf{428571}428571428571\ldots$ is written as $0.\dot{4}28571\dot{}$

Facts to learn

Learn these 23 conversions from fractions to decimals and back again.

Fraction	Decimal
$\frac{1}{4}$	0.25
$\frac{1}{2}$	0.5
$\frac{3}{4}$	0.75
$\frac{1}{5}$	0.2
$\frac{2}{5}$	0.4
$\frac{3}{5}$	0.6

Fraction	Decimal
$\frac{4}{5}$	0.8
$\frac{1}{9}$	0.1
$\frac{2}{9}$	0.2
$\frac{4}{9}$	0.4
$\frac{5}{9}$	0.5
$\frac{7}{9}$	0.7

Fraction	Decimal
$\frac{8}{9}$	0.8
$\frac{1}{10}$	0.2
$\frac{3}{10}$	0.3
$\frac{7}{10}$	0.7
$\frac{9}{10}$	0.9
$\frac{1}{3}$	0.3

Fraction	Decimal
$\frac{2}{3}$	0.6
$\frac{1}{8}$	0.125
$\frac{3}{8}$	0.375
$\frac{5}{8}$	0.625
$\frac{7}{8}$	0.875

Comparing the sizes of decimals

302 is *greater* than 32, but 3.02 is *less* than 3.2

This is because $3.02 = 3 + \frac{2}{100}$ and $3.2 = 3 + \frac{2}{10}$, $\frac{2}{10} > \frac{2}{100}$

A good strategy to use when comparing decimals is to add zeros so they have the same number of decimal places.

Example

Q. Rank these decimals from smallest to largest:

1.001, 0.08, 0.2, 0.09, 0.008, 0.1, 0.11

A. Write the decimals to 3 dp (thousandths).

1.001, 0.080, 0.200, 0.090, 0.008, 0.100, 0.110 [add extra zeros to give 3 decimal places]

$\frac{1001}{1000}, \frac{80}{1000}, \frac{200}{1000}, \frac{90}{1000}, \frac{8}{1000}, \frac{100}{1000}, \frac{110}{1000}$ [write as fractions]

Rank the decimal by comparing the numerators of each fraction.

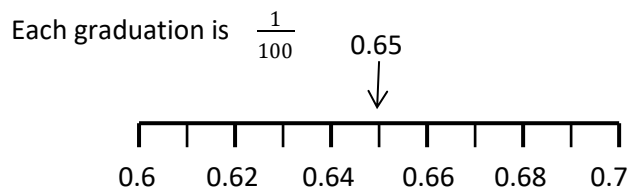
From smallest to largest is: 0.008, 0.08, 0.09, 0.1, 0.11, 0.2, 1.001.

Sometimes, using a **number line** is an easier strategy.

Example

Q. Find a decimal half way between 0.6 and 0.7

A. Use a number line between 0.6 and 0.7 that has 10 graduations.



The arrow shows that halfway between 0.6 and 0.7 is 0.65 *Or*

The mean of 0.6 and 0.7 is $\frac{0.6+0.7}{2} = 0.65$, which is halfway between them.

Activity 3.3

Two kinds of decimals and comparing sizes of decimals

1. State whether the following fractions, when converted, are either terminating or recurring decimals.

a. $\frac{2}{5}$ b. $\frac{2}{3}$ c. $\frac{1}{7}$ d. $\frac{13}{50}$ e. $\frac{5}{11}$

2. Change the following fractions to recurring decimals using appropriate dots over digits.

a. $\frac{7}{9}$ b. $\frac{2}{9}$ c. $\frac{11}{6}$ d. $\frac{11}{13}$ e. $\frac{1}{33}$

3. Write down a decimal that lies *halfway* between each of the following.

a. 324, 325 b. 0.14, 0.18 c. 0.2, 0.3 d. 1.09, 1.1

4. Select two equal decimals form this list: 1.47, 1.047, 10.47, 0.1470, 1.4070

5. Rank the following decimals from smallest to largest.

a. 1.72, 2.0, 0.99, 2.1, 1.80, 1.9 b. 0.40, 0.07, 0.11, 0.99, 1.01, 1.01

6. Rank the following numbers from smallest to largest.

$\frac{19}{10}, \frac{6}{5}$,1.5, 0.95, $\frac{12}{100}, \frac{18}{200}$, 1

Rounding decimals

Decimal places (dp)

It is not always practical or necessary to use all the decimal places obtained from a decimal calculation. Consider a length of wood of 26 cm cut into 3 equal lengths. 26 ÷ 3 = 8.666 66…cm.

It is not appropriate to say the length of each piece is $8\frac{2}{3}$ or 8.6 cm.

An accuracy of 1 dp in centimeters is more appropriate. This is the same as saying the length is accurate to the nearest millimeter.

Rounding a decimal to 1 dp

When **rounding** a decimal to 1 dp:

- Draw an arrow between the 1st and the 2nd decimal place.
- If the 2nd dp is 5 or more, add 1 onto the 1st dp. Delete digits to the right of the arrow.
- If the 2nd dp is less than 5, leave the 1st dp as it is. Delete digits to the right of the arrow

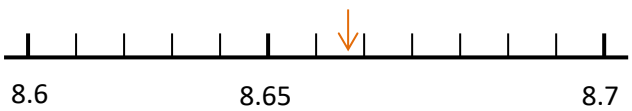
A similar process works for rounding to 2 dp, and higher.

Example

Q. Express 8.666 666… cm to 1 decimal place (nearest mm).

A. The rounded answer is between 8.6 and 8.7

The second decimal place, 6, makes the answer closer to 8.7 than to 8.6



So, 8.666 666…cm is rounded to 8.7 cm (1 dp) Or, imagine 8.666 666…separated by an arrow after 1 dp as 8.6 66 66…The digit after the arrow is greater than, so the number is closer to 8.7 than 8.6.

Significant figures (sig.fig or sf)

Decimal have a certain number of significant figures. It is sometimes more appropriate to round decimals to a given number of significant figures (sf).

The first non-zero digit in a number is the first significant figure. All digits after this, including zeros, are significant.

Zeros at the end of a whole number are *not* significant.	For 305, 30.5 and 3.50, each zero is significant.
Zeros at the end of a decimal *are* significant	For 036, 0.6, 0.0436 and 023.1, none of the zeros is significant.

Examples

Q. Round the following to the number of significant figures given.

a. 247 to 2 sf b. 26.748 to 3 sf c. 0.0453 to 2 sf d. 295 to 2 sf.

A. **a.** Place arrow after 2 significant figures: 24↓7

The 3rd figure is 7, which is 5 or more, so round up to give 25.

A zero is required as a placeholder.

Or just note that 247 is closer to 250 than 240.

So, 247 = 250 (2 sf)

b. Place arrow after 3 significant figures: 26.7 ↓48
The 4th figure is 4, which is less than 5, so leave the 3rd figure as it is

26.748 = 26.7 (3 sf)

c. Place arrow after 2 significant figures: 0.045 ↓3
The first significant figure is 4. The 3rd figure is 3, which is less than 5, so leave 2nd figure as it is.

0.0453 = 0.045 (2 sf)

d. Place arrow after 2 significant figures: 29 ↓5
The 3rd figures is 5, which is 5 or more, so add 1 onto the 2nd figure to give 30
A zero must be included as a placeholder to make the rounded figure similar in size to the original.
295 = 300 (2 sf)

Note: *When answering a question it is usual to round answers to the accuracy of the least accurate decimal in the original numbers*

Identifying rounding

When a decimal has been rounded to a given number of decimal places or significant figures, the rounding can be identified 3.24 has been rounded to 2 dp or 3 sf.

The number 3 9000 could indicate 2 sf, 3 sf, or even 4 sf.

A rounded number like 3.20 incudes the zero because the rounding was 2 dp, or 3 sf.

Activity 3.4

Rounding decimals

1. Choose the correct answer from the numbers in the brackets.
   a. 8.989 to 1 dp (8.9, 8.10, 8.99, 9.0)
   b. 90994 to 2 sf (9.0, 10.9, 100, 10.0)
   c. 210 to 2 sf (21, 220, 200, 210)
   d. 0.09 to 1 dp (0.1, 0.0, 0.01, 1.0)
   e. 500.9 to 3 sf (500, 500.0, 501, 501.0)
   f. 4.948 9 to 1 dp (4.9, 5.0, 5.1, 5.01)
2. Round the following to the decimal places given:
   a. 0.068, 2 dp b. 2.456 7, 3 dp c. 0.3361, 3 dp
   d. 1.004 9, 2 dp e. 5.98, 1 dp f. 4.000 98, 2 dp
3. Round the following to the significant figures given:
   a. 0.0685, 2 sf b. 28.3, 2 sf c. 330.82, 3 sf d. 300, 2 sf
4. Use a calculator to work out the answers to the following. Round sensibly, stating the decimal places used
   a. 3.4×1.92 b. 24.06×0.2 c. 157.1×13.025
   d. $0.03 \div 1.82$ e. $1.07 \div 0.8$ f. $12.142 \div 1.03$
4. State the number of decimal places the following answers have been rounded to.
   a. 1.0 b. 12.07 c. 120 d. 10 200
5. State the number of significant figures to which that the following answers have been rounded.
   a. 369 b. 12.07 c. 120 d. 10 200
   e. 0.008 f. 121.0 g. 2

Calculating with decimals

The operations of adding, subtracting, multiplying and diving decimals without using a calculator are required to be known.

Being able to apply mental strategies and numeracy skills learnt in the past to problems involving small numbers is expected at this level.

Addition and subtraction of decimals

An algorithm for adding and subtracting decimals is to line up the decimal points of each of the numbers and then add (or subtract) as if they are whole numbers.

Examples

Q. Use an algorithm to calculate 34.05 + 2.3 + 1.209

A. Align decimal points so digits with same place value can be added to get 37.559.

$$\begin{array}{r} 34.05 \\ 2.3 \\ +1.209 \\ \hline 37.599 \end{array}$$

Q. Use algorithm to calculate 16.04 – 3.274.

A. Align decimal points so digits with the same place value can be subtracted, to get 12.766. It is easier to subtract if a space is filled with a zero.

$$\begin{array}{r} 16.04\ 0 \\ -\ 3.274 \\ \hline 12.766 \end{array}$$

Remember, other strategies can be used to simplify the calculation.

Example

Q. 24.55 + 3.2

A. Add24.55 to 3.2.

The problem becomes 24 + 3.75 = 27.75

Multiplication of decimals

When multiplying two decimals:

- ignore the decimal point and multiply the resulting whole numbers
- count the total number of decimal places in the two decimals
- insert the decimal point into the answer to five the same total number of decimal places you have counted.

Example

Q. 1.4 × 0.02

A. 14 × 2 = 28 [multiplying without decimal points]

1 + 2 = 3 [decimal places in 1.4 and 0.02]

Therefore, there needs to be 3 decimal places in the answer.

So 1.4 × 0.02 = 0.028 [inserting place holder zero]

Are you using your calculator to multiply decimals? Try first using the numeracy strategies you have learnt in previous years.

Division of decimals

To divide decimals, always divide by a whole number.

The decimal point in the answer is placed directly above the decimal point in the question.

Example

Q. $24.81 \div 6$

A. $6\overline{)24.8^{2}1^{3}0}$ $\quad 4.13$ [insert zeros until there is no remainder or the answer is a recurring decimal]

The answer is 4.135

When dividing one decimal by another decimal, multiply each number by a power of 10 so division is by a whole number.

Example

Q. $2.481 \div 0.06$

A. Consider the division in fraction form.

$$2.481 \div 0.06 = \frac{2.481}{0.06}$$

$$\frac{2.481}{0.06} \times \frac{100}{100} = \frac{248.1}{6}$$ [× 100 makes the denominator a whole number]

So, $2.481 \div 0.06 = 2.481 \div 6$ [using working above]

$= 41.35$

Multiplying by powers of 10

When multiplying *whole numbers* by **powers of 10**, zeros are inserted to the right of the number:

e.g. $37 \times 10^3 = 37\,000$

Consider each digit's place value:

- the 3 in the tens column has moved to the ten thousands column
- the 7 in the ones column has moved to the thousands column.

So each digit has a place value 1 000 times greater than before.

When multiplying decimals by powers of 10 the digits all move to a higher place value. The decimal point appears to move to the right by the same number places as there are zeros in the power of 10.

Examples

Q. a. 2.1×10^3 b. 0.01054×10^4

A. a. $2.1 \times 10^3 = 2.1000 \times 10^3$ [insert extra zeros to give enough place values]

$= 2\,100.0$ [each number moves up 3 place values, so the decimal point moves 3 places to the right]

$= 2\,100$ [decimal point does not need to be included]

b. $0.01054 \times 10^4 = 105.4$ [move the decimal point 4 places right; remove leading zeros]

Dividing by powers of 10

Dividing decimals by powers of 10 is the opposite of multiplying by powers of 10.
Each digit in the decimal will move to a smaller place value.
This means the decimal point appears to move to the left the same number of places as there are zeros in the power of 10.

Example

Q. $\frac{3}{10^4}$

A. $\frac{3}{10^4} = \frac{3.0}{10^4}$ [insert a decimal point]

$= \frac{00003.0}{10^4}$ [insert extra zeros]

$= 0.00030$ [move the decimal point 4 places to the left]

$= 0.0003$ [leave out the unnecessary zero at the end]

Activity 3.5

Calculating with decimals

This exercise is to be done without the use of a calculator.

1. a. 3.08 + 15.51 b. 0.25 + 0.025 c. 7 − 1.09 d. 4.8 + 5.01 + 0.09
   e. 1.4 − 0.7 + 0.06 f. 6.8 − 1.04 − 3.07
2. a. 14 × 0.3 b. 0.024 × 6 c. 9.08 × 0.01 d. 1.2 × 0.3 × 0.2
   e. 63.9 ÷ 9 f. 1.21 ÷ 1.1
3. Find the missing numbers:
   a. 0.3 × _____ = 0.06 b. 0.02 + ____ = 0.22
   c. 0.5 _____ = 0.47 d. 0.2 ÷ _____ = 0.04
4. Given that 207 × 17 = 3 519, write the answer to the following without using the calculator.
   a. 207 × 1.7 b. 0.207 × 17 c. 20.7 × 0.017
5. Use the correct order of operation (BEDMAS) to work out the following.
   a. 1.03 + 0.7 x 2.02 b. $\frac{1.08\times0.2}{3+0.2}$ c. $\frac{0.4}{0.02} + \frac{1.2}{0.4}$

Problem solving with decimals

In practical problems involving decimals, decide which of the operations +,−,× or ÷ are required to solve the problem. Some solutions may involve several different operations. Answer the problem with a sentence, including units and any rounding.

Example

Q. A packing case weighs 2.4 kg when empty. Inside are 20 packets of soap powder, each weighing 0.7 kg.

a. What is the total weight of a full packing case of the soap powder?
b. How many full packing cases can be loaded onto a truck that can carry no more than 2 000 kg?

A. a. Total weight = 2.4 + 20 × 0.7 [packing case + 20 packets of soap]

= 16.4

Total weight of a full packing case is 16.4 kg

b. 2 000 ÷ 16.4 = 121.95 (2 dp) [maximum load÷one packet weight] The number of full packing cases is 121 cases.

(Only whole numbers of cartons of soap can be loaded onto the truck, and the truck load weight cannot be exceeded, so round *down* to the nearest whole

Activity 3.6

Problem solving with decimals

1. A trip in a taxi costs a basic charge of $4.50 and then $3.25 for every km travelled. Find the cost of a trip of 22 km.
2. Oliver worked out he required 7.75 m^2 of tile to make a BBQ patio. If he already has 2.90 m^2 of tiles, what area of tiles does he need to buy?
3. Maia saw a tracksuit costing $ 157.59 and decided to buy it on hire purchase. The additional charges were $54.26 interest, $5.50 insurance and an administration fee of $6.20. How much does Maia have left to pay after she makes a down payment of $25?
4. Alisi has a piece of wood measuring 10.55 m. It needs to be cut into four equal lengths. How long is each piece of wood, to 2 dp?
5. There were six athletes in the final of the Year 10 high jump. The judge said the mean height was 1.73 m. If Hema, who had jumped 1.88 m, was disqualified, what would be the new mean height of the five athletes?
6. In a team race of 20.5 km, there are 7 runners involved.
   a. What should be the distance for each athlete to run if they are given equal distance? (Round your answer to 3 dp.)
   b. If it is decided that the first six runners would each run 2.85 km, what would be the distance run by the last runner?
7. Aina has $55.60 and Susana has $24.90. How much money will Aina give to Susana if they then each have the same amount of money?
8. In an examination, it is known that 0.55 of the students failed. If 81 students passed, how many took the exam?

Standard form

Standard form, sometimes called **scientific notation**, is a convenient way to write very large or very small numbers.

In standard form, a number is written in the form $a \times 10^n$, where n is an integer, and $1 \leq a < 10$ (a is a number between 1 and 10, a can equal 1, but cannot equal 10.)

For very large numbers, n is positive.

Science often deals with very large or very small numbers. A 'light year' is approximately equal to 9 500 000 000 000 km. This form takes up a lot of room and cannot be easily compared with other numbers of a similar size!

Example

Q. Write 9 500 000 000 000 in standard form.

A. Place the decimal point after the first significant digit so that the number lies between 1 and 10, so a = 9.5

$$9\,500\,000\,000\,000 = 9.5 \times 1\,000\,000\,000\,000$$
$$= 9.5 \times 10^{12} \quad \text{[12 zeros so index, n = 12]}$$

The number in standard form is 9.5×10^{12}

Division by 10^n is the same as multiplication by 10^{-n}, so for very small numbers in standard form, n is negative.

Example

Q. Write 0.000 000 004 87 in standard form.

A. 0.000 000 004 87 = 4.87÷ 1 000 000 000 [4.87 is between 1 and 10]

= 4.87÷ 10^9

= 4.87÷ 10^9 [9 zeros, so n = 9]

= 4.87× 10^{-9} [negative index changes ÷ into ×]

The number in standard form is 4.87 × 10^{-9}

Know your calculator. Standard form problems can be done using a scientific calculator.

Different calculators use different keys and methods

When comparing the sizes of numbers in standard form the number with the highest power of 10 is largest.

If numbers written in standard form have equal powers of ten, then the number with the highest decimal is largest.

Example

Q. Arrange the following numbers written in standard form from greatest to smallest:

3.9 × 10^3, 1.2 × 10^4,6.8 × 10^3

A. 1.2 × 10^4 is largest [highest power of 10]

6.8 > 3.9, so 6.8 × 10^3 is greater than 3.9 × 10^3 [equal powers of 10 so compare decimals]

In descending order, the numbers are:

1.2 × 10^4, 6.8 × 10^3,3.9 × 10^3

When solving problems with numbers in standard form, separate out the powers of 10.

Index laws will be used here: $10^n \times 10^m = 10^{n+m}$,$10^n \div 10^m = 10^{n-m}$

Examples

Q. Calculate 4.1 × 10^6 multiplied by 7.5 × 10^5

A. 4.1 × 10^6 × 7.5 × 10^5

= (4.1 × 7.5) × (10^6 × 10^5) [partition numbers in like form]

= 30.75 × 10^{11} [multiply by adding the powers of 10]

= 3.07 × 10^1 × 10^{11} [writing 30.75 in standard form]

= 3.075 × 10^{12} [adding powers of 10]

Q. Calculate $\frac{2.4\times10^7}{1.25\times10^4}$

A. $(2.4 \div 1.25) \times (10^3 \div 10^4)$

$= 1.29 \times 10^3$ [subtracting powers of 10]

Activity 3.7

Standard form

1. Write in standard form:
   a. 423 000 b. 75 c. 287.4 d. 1 million
   e. 0.5×10^7 f 165×10^3 g. 0.002 h. 0.7
2. Write as normal decimals:
   a. 1.3×10^4 b. 4×10^7 c. 9.2×10^4 d. 3.8×10^{-5}
   e. 1.07×10^{-1} f. 9.9×10^{-7}
3. a. Write the answer to $2\,005 \times 18\,000$ in standard form.
   b. Write the answer to $0.2 \times 18\,000$ in standard form.
   c. Write the answer to $26.2 \div 4\,075$ in standard form.
4. The greatest distance of the Moon from Earth is 405 470 km.
   a. Write this distance in standard form.
   b. Write your answer in standard form to 3 significant figures.
5. Which is the largest of the numbers: A 6.8×10^{-4} or B 1.3×10^{-3} or C 9.2×10^{-5}
6. Work out the following, leaving your answer in standard form:
   a. $(2.07 \times 10^4) \times (3.1 \times 10^2)$ b. $(6.07 \times 10^4) \times (9.1 \times 10^2)$
   c. $(5.07 \times 10^4) \times (2.1 \times 10^2)$ d. $(2.07 \times 10^4) + 3.1 \times 10^4$