Strand: Number

Substrand: Percentages

- Find percentage of a quantity
- Express one quantity as percentage of another
- Increase or decrease by a percentage
- Inverse percentage increase /decrease problems (finding the original amount)
- Solve familiar problems that involves percentage
- Convert percentage to decimal or fraction
- Convert decimal to fraction or percentage
- Convert fraction to decimal or percentage

Revising percentages

The abbreviation of percentage is *per cent*, or %.

Percentage means: 'Out of 100': e.g. 85% means 85 out of 100

The % symbol is made up by rearranging a 1 and 00.

Percentage conversions

Fractions and **decimals** can be converted to percentages, and vice versa.

$$\text{fractions} \times 100 = \text{percentage} \qquad \text{decimal} \times 100 = \text{percentage}$$

$$\frac{\text{percentage}}{100} = \text{fraction} \qquad \frac{\text{percentage}}{100} = \text{decimal}$$

Examples

Q. Convert $\frac{3}{40}$ to a percentage.

A. $\frac{3}{40} \times 100 = \frac{300}{40} = 7\frac{1}{2}\%$ [fraction × 100 = percentage]

Q. Convert 3% to a decimal.

A. $3\% = 3 \div 100 = 0.03$ [percentage ÷ 100 = decimal]

Q. Convert 65% to a fraction.

A. $65\% = \frac{65}{100}$ $\left[\frac{\text{percentage}}{100} = \text{fraction}\right]$

$= \frac{13}{20}$ [fraction is simplified]

Improper fractions or **mixed numbers** can be expressed as a %

Example

Q. Express $\frac{8}{5}$ as a percentage.

A. $\frac{8}{5} = \frac{8}{5} \times \frac{20}{20}$ [this fraction converts easily to a fraction out of 100]

$= \frac{160}{100}$

$= 160\%$

It is useful to memorise conversions of commonly used percentages.

Percentage	Decimal	Fraction
5%	0.05	$\frac{1}{20}$
10%	0.1	$\frac{1}{10}$
$12\frac{1}{2}$%	0.125	$\frac{1}{8}$
20%	0.2	$\frac{1}{5}$
25%	0.25	$\frac{1}{4}$
$33\frac{1}{3}$ %	$0.\dot{3}$	$\frac{1}{3}$
50%	0.5	$\frac{1}{2}$
$66\frac{2}{3}$ %	$0.\dot{6}$	$\frac{2}{3}$
75%	0.75	$\frac{3}{4}$
100%	1	1

Examples

Q. About 85% of an iceberg is below the surface of the sea. Write this percentage as a decimal and as a fraction.

A. 85 ÷ 100 = 0.85

0.85 of an iceberg is below the surface of the sea.

$\frac{85}{100} = \frac{17}{20}$ [simplify the fraction]

$\frac{17}{20}$ of an iceberg is below the surface of the sea.

Q. Luke has the results of his test marks in English and Maths. He scored 64 out of 80 in English and 42 out of 50 in Maths. Which is the better mark?

A. % in English is $\frac{64}{80} \times 100 = 80\%$

% in Maths is $\frac{42}{50} \times 100 = 84\%$

The better mark is Maths.

Activity 4.1

Revising percentages

1. Copy and complete the table of conversions. Write fractions in their simplest form.

	Fraction	Decimal	Percentage
a.	$\frac{5}{8}$		
b.		0.45	
c.			150%
d.		0.025	
e.	$\frac{9}{40}$		
f.			1%
g.	$\frac{13}{4}$		
h.		2.05	
i.			0.5%
j.	$\frac{12}{300}$		

2. Fita wins some money on Lotto and decides to save 85% of it.
   a. What fraction of his winnings does he save?
   b. What fraction of his winnings does he spend?
3. After Lisa had walked $\frac{9}{20}$ of her journey, she stopped for a rest. What percentage of the journey had she completed?
4. Which is the biggest:
   0.476, 47%, or $\frac{12}{25}$?
5. Express 0.3% as a decimal and as a fraction.
6. Write in order from least to greatest: 30%, $\frac{3}{5}$, 0.24.

Express one quantity as a percentage of another

To express one quantity as a percentage of a second quantity:

$$\text{Percentage} = \frac{1st\ quantity}{2nd\ quantity} \times 100\%$$

Example

Q. Express 30 as a percentage of 240

A. Percentage = $\frac{30}{240} \times 100 = 12.5\%$ [using calculator]

Quantities being compared must be expressed using the same units.

Example

Q. Express 45 g as a percentage of 2 kg

A. 2 kg = 2 000 g [converting to same units]

percentage = $\frac{45}{2\ 000} \times 100 = 2.25\%$

Percentage of a quantity

Finding a percentage amount of a quantity

To find the percentage of a quantity, multiply by the percentage.

Examples

Q. What is 12% of 60?

A. 12% of 60 = 12% × 60 [multiply by percentage]
= 0.12 × 60 [changing 12% into a decimal]
= 7.2

Q. A rugby game lasts for 80 minutes and it is decided to extend the time by 15% because of time off for injuries. How much extra time is allowed?

A. 80 × 15% = 80 × 0.15 [quantity multiplied by percentage]
= 12
The extra time is 12 minutes.

Finding a quantity when a percentage is given

To find the percentage of a quantity, divide by the percentage.

Examples

Q. 24 is 15% of what number?

A. 24 ÷ 15% = 24 ÷ 0.15 [dividing quantity by the %]
. = 160

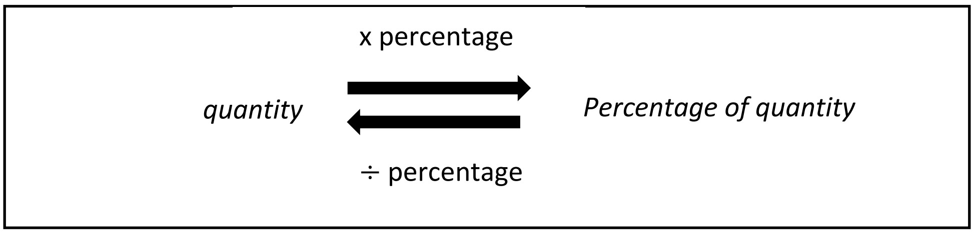
Q. When the exams are due, Albert decides to increase the time he spends on homework per night by 54 minutes. This is 60% of his usual time. What is the usual time he spends on homework?

A. The increased time is 54 minutes and the percentage is 60%

54 ÷ 60% = 54 ÷ 0.6 [dividing quantity by the %]

= 90
Usual time on homework is 90 minutes.

To find a quantity when the percentage is known is the reverse process of finding the percentage of a quantity:



Examples

Q. Lisi needs a new bike. The price of the bike she wants is $680, and the deposit is 22%. How much does she need to pay for the deposit?

A. Find 22% of $680

$680 × 22% = 680 × 0.22 [quantity multiplied by percentage]

= 149.6
Lisi pays $149.60 as the deposit.

Q. Lisi decides to buy a new bike. She is asked to pay a deposit of $149.60 which is 22% of the price of the bike. What is the price of the bike?

A. 22% of the price of the bike is $149.60

149.60 ÷ 22% = 149.60 ÷ 0.22 [quantity divided by the %]
= 680

The price of the bike is $680.

Activity 4.2

Percentage of quantities

1. Express the first quantity as a percentage of the second quantity.
   a. 72 seconds, 2 minutes b. 4 hours, 1 day c. 200 m, 1 km
   d. 1 000 mL, 1 L e. 500 mm, 10 cm
2. Find:
   a. 10% of 8 m b. 25% of 17 L c. 0.5% of $600
   d. 100% of 0.26 cm e. 150% of 12 f. 67.5% of 80 hectares
   g. 20% of 324.5 cm^2 h. 66% of 390 km.
3. In each question in the following table, the first quantity is a percentage of the second quantity. Copy and complete the table.

	1st quantity	2nd quantity	%
a.	18		75%
b.		9	$33\frac{1}{3}\%$
c.	12		200%
d.		200	0.5%

4. The following table contains information about the number and types of home built by a builder over a period of three years.

	2010	2011	2012
Detached houses	24	31	45
Townhouses	16	13	10

   a. What percentage of these townhouses did he build in the year 2010?
   b. What percentage of all these houses were townhouses?
   c. In the year 2012, What percentage of the houses built were townhouses?
5. At a dinner party for six people, the host decides to buy 150 g of steak of each person. When meat is cooked there is a 25% loss in weight due to evaporation. What is the total weight of meat lost due to evaporation?
6. The Performing Arts Centre at a school holds 450 when full. How many people attended a performance of 'Madam Butterfly' if the auditorium was 82% full?

Increasing or decreasing a quantity by a percentage

When a quantity changes, it either increases or decreases as a percentage of the *original quantity.*

Consider $300 increased by 50%.
50% of $300 is $150, so $300 + $150 = $450
Look at this problem in another way.
$300 is 100% of the original quantity
So, $450 would now be 100% + 50% = 150% of the original quantity

150% of \$300 = 1.5 × \$300 = \$450
1.5 is the *multiplying* factor on the \$300

When quantity is increased by r %, (100 + r) % is the multiplying factor:

original quantity × (100 + r) % = final quantity

If \$300 is *decreased* by 50%, then the final quantity is \$300 − \$150 = \$150

The final quantity \$150 is 100% − 50% = 50% of the original quantity

0.5 is now the *multiplying* factor on the \$300

When a quantity is decreased by r %, (100 − r) % is the multiplying factor.

Original quantity × (100 – r)% = Final quantity

Examples

Q. Siona weighs 95 kg and decides to go on a diet to reduce weight. The doctor tells him to reduce his weight by 15 %. What is Siona's target weight?

A. This problem is about decreasing Siona's weight, so the multiplying factor is (100 − 05)% = 85%,

Siona's new weight = original weight × 85 %,
= 95 × 0.85 %, [85 % = 0.85]
= 80.75

Siona's target weight is 80.75 kg

Q. 12 000 copies of a new DVD were sold in the first week of its release. There was a 22 % increase in sales in the second week. What were the sales in the second week?

A. This problem is about *increasing* sales, so the multiplying factor is

(100 + 22) % = 122%

Copies sold in second week = original sales × 122 %,
= 12 000 × 1.22 [122 % = 1.22]
= 14 640

The number of DVDs sold in the second week is 14 640 copies.

Inverse percentage increase/decrease problems (finding original amounts)

A percentage increase or decrease is always of the original amount.
To find the original quantity when it has been increased/decreased by a percentage reverse the process for finding the final quantity:

Final quantity ÷ multiplying factor = Original quantity

Example

For problems in which the original quantity is increased, the multiplying factor will be greater than 100%, and for a decreased quantity it will be less than 100%.

Q. Holi goes on a diet to lose weights after her doctor Advised her to reduce her weights by 12 %, As a result of a successful diet, Holi now weight 72 kg. What was her weight before she diets?

Check for a sensible answer. Holi lost weight, so the answer should be a larger amount than 72 kg

A. Holi has a lost weight, so this is a decrease. The multiplying factor is (100 − 12) % = 88%
[subtract 12 % from 100 %]
final weight ÷ multiplying factor = original weight.
72 ÷ 88 % = 72 ÷ 0.88 = original weight [88 % = 0.88]

Activity 4.3

Increasing and decreasing by a percentage

1. Increase the following quantities by the percentage given.
   a. 40 L by 50% b. 19 km by 100% c. 10 tonnes by 0.5%
   d. 50 L by 50% e. 47.5 kg by 16.4% f. 42.5 m^2 by 2.5%
2. Decrease the following quantities by the percentage given.
   a. $56 by 40% b. 200 km by 50% c. 150 L by 0.75%
   d. 158 tonnes by 15% e. 4.8 kg by 25% f. 15.9 m^3 by 12.5%
3. The price of a cookbook is reduces by 15%. If the discount is $5.40, what is the original price of the book?
4. Tevita spends 90% of his income and saves the rest. If he saves $90 per week, what is his weekly income?
5. Fano has just bought a car for $15 000. It will depreciate in value at a rate of 30% in the first year and 15% in the second year.
   a. What will her car be worth at the end of the first year?
   b. What will her car be worth at the end of the second year?
   c. Why is the answer to a. not the same as a single depreciation of (30 + 15) = 45%?
6. An article was sold for $100, which include a profit of 100% What is its cost before the profit was added on?
7. A man received a pay rise of 15% and his new pay rate was $34.50 per hour. What was his old pay rate?

Financial Literacy

Consumer maths

Learning Outcomes:

- Determine better price of an item given its discount amount in dollars and in percentage
- Solve practical problems of money amount
- Determine differences in cost of goods
- Use the unit prices of goods to determine total cost or amount of change from a shopping

Consumption Tax (CT)

Consumption Tax (**CT**) is a government tax added to the price at which goods or services are sold.

- Before CT is added to the price of goods, the price is *exclusive* of CT, *or* CT *excluded*.
- After CT has been added, the price is *inclusive of CT, or CT-included*.

In Tonga the CT rate is currently 15%

The CT-included price is 115% of the CT-excluded price, so:

price excluding CT × 1.15 = price including CT
price including CT ÷ 1.15 = price excluding CT

Examples

Q. Pita owns a paint and decorating business and has a customer enquiry about painting a bathroom. His quote is $550 exclusive of CT. How much CT will have to be added to the cost?

A. CT is 15% of the price that is exclusive of CT
CT = 550 × 0.15 [15% = 0.15]
= 82.5
CT is $82.50. This will be added to the quote of $550 when payment is made by the customer.

Q. Before CT of 15% is added, a hat costs $36.00. What is the price of the hat after the CT is included?

A. 36 × 1.15 = 41.4 [price excluding CT × 1.15 = price including CT]
The CT-inclusive price of the hat is $41.40

Q. Sesi buys a bike for $253, which includes CT at 15%. What was the price of the bike before CT was added?

A. 253 ÷ 1.15 = 220 [price including CT÷ 1.15 =price excluding CT]
The price of the bike exclusive of CT is $220

Activity 4.4

CT

1. Find the CT on the following prices that do not include CT. (These are basic questions which can be done using a mental strategy of finding 10% of the price and adding on half of this to obtain 15%)

a. $100 b. $60 c. $200 d. $600 e. $120

2. Find the CT on the following prices that do not include CT. (Where appropriate, round answers to the nearest 10 cents)

a. $170 b. $611 c. $19.99 d. $683 e. $120.50

3. The prices in this question are exclusive of CT. Find the prices when CT has been added. (Where appropriate, round answers to the nearest 10 cents.)

a. $26 b. $124 c. $250 d. $47.50 e. $59.99

4. The following prices include CT. Find each price when CT has been removed. (Where appropriate, round answer to the nearest 10 cents.)

a. $418 b. $655 c. $1.99 d. $69.99 e. $100

5. The price, including CT, of a TV is $2 500. Mikaela, the manager, decides to reduce the CT-exclusive price in a sale by 25%. What is the sale price of the TV when CT is included?
6. The selling price of a dress is $120, before CT is added. What is the price of the dress after CT is included?
7. Longani sells foods in his shop and has decided that his customers will never pay more than $10 for any item. What is the highest price he can charge for an item before CT is added?

Applications of percentages

Business applications use the following vocabulary.

- *retailer* – a shopkeeper
- *cost price* – the price at which a retailer buys stock
- *selling price* – the price at which a retailer sells stock
- *profit* – the amount added to a cost price to arrive at a selling price (or mark-up)
- *percentage profit* – profit expressed as a % of the cost price

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

- *discount* – amount subtracted from a selling price; discounts are often expressed as a percentage of the selling price.

Examples

You can use mental strategies for straightforward problems: $150 is $\frac{1}{2}$ of $300, so profit is 50%

Q. A retailer buys a cycle for $150 and sells it for $300. What is his percentage profit?

A. Profit = 300 − 150 [profit = selling price – cost price]
= $150

% profit = $\frac{150}{300} \times 100$ [% profit = $\frac{\text{loss}}{\text{original price}} \times 100$]

The percentage profit is 50%

This problem can also be done mentally. Did you notice that $\frac{40}{200} = \frac{20}{100}$
So, discount = 20%

Q. A furniture shop has some old stock of coffee tables, which normally sell for $200 each. If the shopkeeper sells them for $160 each, what is the percentage discount?

A. Discount = 200 − 16 = $40

% discount = $\frac{40}{200} \times 100$ [% loss = $\frac{\text{loss}}{\text{original price}}$ × 100]

The percentage discount is 20%

Q. Siniva buys a netball from the supplier for $38. How much should she sell it for to make a profit of 64%?

A. To find the price after the percentage increase, use the multiplying factor is

The multiplying factor is (100 + 64) % = 164 %

new price = 38 × 164%

= 34 × 1.64 [164% = 1.64]

= 62.32

Siniva should sell the netball for $62.32

Simple and Compound interests

Learning Outcomes:

- Define the simple interests and compound interests of an investment or loans
- Differentiate between simple and compound interests
- Calculate the simple interest for investments and loans
- Calculate the compound interest for investments and loans
- Solve problems involving compound interest
- Find the Depreciation and successive discount
- Evaluate best buy and special offers. e.g., discounts
- Apply financial literacy in solving related problems in context

Interest

Interest is money paid for the use of money (the **principal**). Interest is expressed as a percentage rate. **Simple interest** is worked out using the formula:

$$\text{Simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

Example

The simple interest on \$400 at 3% for 2 years is

Simple interest = $400 \times 0.03 \times 2 = \$24$

Compound interest includes the interest with the principal at the end of each **period** (time over which the interest is calculated).

Example

\$400 is invested at 3% compound interest per year for 2 years. At end of year 1, investment is worth:

$400 \times 1.03 = \$412$ [103% of previous value]

At end of year 2, investment is worth:

$412 \times 1.03 = \$424.36$

Total interest = $424.36 - 400 = \$24.36$

Note: *A single* ***calculation of*** $400 \times 1.03 \times 1.03$ *(or* 400×1.03^2*) will give the value of the investment after 2 years. (The value of the investment after 5 years would be* $400 \times 1.03^5 =$ \$463.71*)*

Purchase options

Different methods of payment are available for buying goods

Bank credit card

The goods are available immediately for use. Money owning on the credit card is paid at a later date.

Hire purchase (HP)

Pay cash deposit (e.g.20% of the price) then pay the rest in regular instalments. The retailer arranges with a finance company to lend the extra money. Extra money is paid in interest.

Cash payment

Save the money and buy with a cash payment. This is the cost of the goods with no interest added. This could take some time.

Bank loan

The bank lends the cash required. Bank is paid back at an agreed sum per week. The interest charged by the bank is usually lower than the interest charged by a finance company.

Activity 4.5

Applications of percentages and Interest

1. Meleliku is buying a new bed on HP. The cash price is $2 100, and she pays a deposit of 10%. The money she owes, which includes an interest rate of 20%, is paid off with 24 equal payments. How much is each payment?
2. Kelepi is buying a cell phone and the cash price is $300. He compares a bank loan with HP method of buying the cell phone.

HP	Bank Loan
10 % deposit, repayments are $28 per month for 12 months.	No deposit, 12 equal repayments that include the $300 an extra charge of 15 % interest.

   Which is cheaper and by how much?
3. Simote works in a hospital and is paid $24.50 per hour. For each hour he works over 40 hours per week, he is paid an overtime rate of time and a half. If Andy pays tax at 30%, what is his take-home pay for a week in which he does 48 hours work?
4. Sifa buys a new TV and the cash price is $740, with an HP deal as follows: 25% deposit, and $34 per month for 18 months. What is the interest rate per annum charged on the remainder after Angus has paid the deposit?
5. Saia invests $500 for 2 years and gets $80 simple interest for the two years. What is the interest rate per annum?
6. A class decides to fund raise by selling sausages. They buy 15 kg of sausages from a local butcher. These sausages usually sell for $6.90 per kg (including CT at 15%) but the butcher has donated the CT content of the price. Other costs include a cylinder of gas for $12 plus GST, tomato sauce at $4.80 and 2 packets of napkins at $3.70 each (purchase at the supermarket).
7. Samiu puts a make-up of 60% on a softball bat, which he then sells for $152. What is the cost price?
8. *Fly-away deals offers*: 'Choose any holiday, pay 10% deposit and pay off the rest with a loan of 5% interest in 12 equal monthly repayments.' Tina chooses a holiday in Queensland at a cash price of $1 800. What is the total cost of her holiday?

Strand: The rates and ratios

Learning Outcomes:

- Define ratios
- Simplify ratios
- Calculate an unknown quantity given the two ratios are equivalent
- Share a quantity into 2 or 3 given ratios and proportions
- Define rates
- . Simplify rates
- Use rates to compare quantities in different units
- Converts rates from one set of units to another
- Apply rates and ratios in solving problems in context

Ratio

A **ratio** is the comparison by division of one quantity with another quantity, using the same units:

- a ratio in which one quantity is twice the size of the other is written as 2:1
- a ratio of two quantities can be expressed as a fraction
- a ratio does not have units
- the numbers in a ratio must be natural numbers, and not fractions or decimals
- the order of a ratio is very important.

Equivalent ratios

Equivalent ratios describe the same comparison.

The ratio 5:10 can be written as the fraction $\frac{5}{10}$ and simplified to $\frac{1}{2}$
So, the ratio 5:10 is equivalent to the ratio 1:2. Both parts of the ratio have been divided by 5.

If the ratio involves fractions or decimals, multiply each part by the same number to simplify to a whole number.

Examples

Q. Find x if the ratio *x*:7 can be written as 25:35

A. $\frac{x}{7} = \frac{25}{35}$ [convert to fraction so knowledge of fractions can be applied]

$\frac{x}{7} \times \frac{5}{5} = \frac{25}{35}$ [re-write as an equivalent fraction with a denominator of 35]

5x = 25 [comparing numerators]

x = 5

Q. The scale of a map is 1:100 000. If the length of a lake is 1.5 km, find its length on the map.

A. 1.5 km = 1 500 m = 150 000 cm [convert km to cm so numbers in ratio have same units]

1:100 000 = ? :150 000 [scale is 1 cm on map to 1000 000 cm in real life]

$$? = \frac{150\,000}{100\,000} = 1.5$$

Length of lake on map is 1.5 cm

Q. The ratio of boys to girls in the Gym Club is 2:3. How many boys are there in the Gym Club if there are 123 girls?

A. 2:3 = number boys: 123 or $\frac{2}{3} = \frac{\text{boys}}{123}$

Remember to keep order boys : girls as in question

$$\frac{2}{3} \times \frac{41}{41} = \frac{\text{boys}}{123}$$

$$\frac{82}{123} = \frac{\text{boys}}{123}$$

So, there are 82 boys in the Gym Club

Activity 4.6

Equivalent ratios

1. Complete the equivalent ratios.
   a. a. 1: a = 6 :18 b. 3: 5 = b:15 c. 7: 35 = 1:c
2. Simplify the following ratios.
   a. 2:4 b. 12:6 c. 10:100
   d. 18:36 e. 24:18 f. 16:56
3. Compare the following pairs of quantities using ratios in their simplest form.
   a. 2 cm and 10 mm b. 2.5 kg and 0.5 kg
   c. $5\frac{1}{4}$ tonnes and $1\frac{3}{4}$ tonnes d. 2 m^2 and 1.5 m^2
4. Write the ratio 12.5: $2\frac{1}{3}$ in its correct form.
5. The length of a belt of trees on a map is 3.5 cm and its real length is 200 m. Express this as a ratio.
6. Ben is mixing concrete. The ratio of cement to sand is 2:13
   a. If Ben has 208 kg of sand, how much cement does he need to make concrete?
   b. If Ben has 54.6 kg of cement, how much sand does he need?
7. There were 32 teams in the World Cup football finals. 20 teams were from Europe, 8 from America, and the rest from Asia. Find the ratios of the number of teams for:
   a. Europe:America b. Total entry:Asia c. America:Asia
8. Simi is saving money and at present he has $ 1 200.
   a. Jim sets a target to increase his savings in the ratio 3:5 from this year to next year. What will be his total savings by the end of next year?
   b. Instead of saving more, Jim spends $150 of his savings of $1 200. What ratio will represent the decrease in his savings between the two years?

Sharing a quantity in a ratio

Dividing quantities by a given ratio can be done in one of two different ways:

- by finding the total number of equal parts
- by using fractions.

Example

Q. Share 333 bananas between three people in proportion to their weights:
John weighs 75 kg, Bill weighs 100 kg and Helen weighs 50 kg.

A. John:Bill:Helen = 75:100:50 [write the weights in a ratio]
= 3:4:2 [simplify the ratio]
3 + 4 + 2 = 9 [add the numbers of the ratio]

This means that the 333 bananas need to be divided into 9 equal parts. John gets 3 parts; Bill gets 4 parts and Helen gets 2 parts.

$333 \div 9 = 37$

This means there are 37 bananas in each of the 9 equal parts.

So: John gets $3 \times 37 = 111$ bananas
Bill gets $4 \times 37 = 148$ bananas
Helen gets $2 \times 37 = 74$ bananas
(Check the answer:111 + 148 + 74 = 333)

Or, for an alternative strategy:

Total number of equal parts = 9

So, John gets $\frac{3}{9}$, Bill gets $\frac{4}{9}$, and Helen gets $\frac{2}{9}$ of the total bananas.

John gets $\frac{3}{9} \times 333 = 111$ bananas
Bill gets $\frac{4}{9} \times 333 = 148$ bananas
Helen gets $\frac{2}{9} \times 333 = 74$ bananas

Q. Tina and Tesi won \$10 000 on Lotto. If Tina was entitled to \$4 000 of the win, how much would you expect her to contribute to the cost of the \$15 ticket?

A. Tina won \$4 000 so Tesi would get the rest, which is \$6 000.

4 000:6 000 = 4:6 = 2:3 [write their winnings as a ratio and simplify]
2 + 3 = 5 [add the numbers in the ratio]

Tina would be expected to pay $\frac{2}{5}$ of the cost of the ticket.

$\frac{2}{5}$ of $15 = \frac{2}{5} \times 15 = 6$

Tina would contribute \$6.00 to the cost of the ticket.

Activity 4.7

Sharing a quantity in a ratio

1. Share \$ 600 in the following ratios.
   a. 17:3 b. 1:4 c. 3:4:5
2. A painter is to mix green and yellow paint in the ratio 4:7 to obtain the colour he wants. If he wants 44 litres of mixed paint, how many litres of yellow paint will he need to mix with the green paint?
3. At a recent Crusaders game, it was estimated the ratio of male to females was 7:3. If there were 12 600 females at the game, what was the total number of spectators at the game?
4. A sum of money is divided between ‘Ele and ‘Olivia in the ratio 5:8. What fraction of the money does ‘Olivia receives?
5. Tala and Hale are saving money. For every \$5 Tala saves, Hale saves \$3. Their target is to save a total of \$1 000. How much does Tala have to save?
6. There are 153 pens in a box. The pens are red, blue or black. The ratio of red to blue to black is 3:2:4. How many red pens are in the box?

7. In a farmyard there are hens, ducks and geese. If of $\frac{1}{5}$ the birds are hens and $\frac{5}{8}$ are ducks:
   a What is the ratio of hens to ducks to geese?
   b If there are 14 geese in the farmyard, how many hens are there?
8. In a box there are 100 pens which are red or blue. The teacher reached in the box without looking and grabbed a handful and found she had 3 red and 7 blue pens. Estimate how many red pens there are in the box.

Increasing or decreasing a quantity in a ratio

A certain quantity can be increased (or decreased) in a given ratio.

Example

Q. Increase a distance of 24 km in the ratio 2:3

A. The distance of 24 km is represented by '2' in the ratio 2:3, and the increased distance is represented by '3':

$$\frac{intial\ distance}{increased\ distance} = \frac{24}{increased\ distance} = \frac{2}{3}$$ [write the distances as a ratio]

The 2 has been multiplied by 12 to get 24, so multiply the 3 by 12 to find the increased distance.

Increased distance = 3 × 12 = 36 km

Q. Uai has saved $50 and then spends $20 of it. In what ratio has his savings decreased?

A. The initial savings is $50 and the final savings is $30, after Uai spent $20.

$$\frac{initial\ savings}{final\ savings} = \frac{50}{30} = \frac{5}{3}$$ → The ratio is 50: 30 = 5: 3

Uai has decreased his savings in the ratio 5:3

Activity 4.8

Increasing or decreasing a quantity in a ratio

1. a. Increase $1 200 in the ratio 4:5
   b. Decrease $1 200 in the ratio 5:2
2. A photocopier is set to reduce the lengths of pictures from 24 cm to 15 cm. What is the ratio of the reduction?
3. Selesia's savings went from $10 000 to $ 6 000. In what ratio has her savings reduced?
4. A photocopier is set to increase lengths in the ratio 5:8
   a. A picture of a cat's height is increased to 20 cm. What was its original height?
   b. What ratio would reduce the cat's height back to its original height?
   c. If the cat's original length was 25 cm, what would be its increased length?
5. When he first went to high school, Siua's height was 1.4m. During his first year his height increased in the ratio 12:13. In his second year, it increased in the ratio 6:7.
   a. Find Siua's height to the nearest cm at the end of his second year.
   b. Siua's friend Daniel had height increases in the same ratios of 12:13 and 6:7 over the first two years at high school. At the end of two years, Daniel's height was 1.9 m. What was his height when he started high school?

Rates

A rate compares one quantity with another quantity, with different units, by dividing the quantities:

E.g. dividing the distance in kilometres by the quantity of time in hours, gives a rate of kilometres per hour.

Other examples of rate in everyday life include a person's heartbeat measured in beats per minute, and fuel consumption on a car trip in kilometres per litre.

Example

Q. David uses 14 L of petrol for a trip of 189 km in his car. How far would he travel on 37 L at the same rate?

A. Rate = $\frac{189}{14}$ [dividing km by L]

= 13.5 km/L

For a trip using 37 L, he would travel 13.5 × 37 = 499.5 km
David would travel 499.5 km on 37 L of petrol.

Rates can be used to compare best buys.

Examples

Q. Which is the better buy, a 3 kg bag of potatoes for $1.55 or a 20 kg sack for $10.60?

A. Calculate the cost per kg:

Rate for the 3 kg bag is $1.55 ÷ 3 kg = $0.52 per kg (2dp)
Rate for 20 kg sack is $10.60 ÷ 20 kg = $0.53 per kg

Q. What is the cost of 1 kg of yoghurt if 125 g costs $1.30?

A. To compare quantities, the same units are required.

125 g costs $1.30 so 1 g costs $1.30 ÷ 125
1 000 g costs $1.30 ÷ 125 × 1 000 = $10.40 [1 kg = 1 000 g]
So, 1 kg yoghurt costs $10.40
Alternatively, multiply cost of 1 kg by 8 (125 × 8 = 1 000).

Activity 4.9

Rates

1. Water empties out of a tank at the rate of 50 litres per minute.
   a. If the tank holds 1 500 litres of water, how many minutes will it take to empty?
   b. How many litres of water does the tank contain if it empties in 5 minutes?
2. The school record for the 800 m is 2 minutes. How fast is this in km/h?
3. If 15 lambs eat a certain number of swedes in 60 days, in how many days will 24 lambs eat the same number of swedes, eating at the same rate?
4. Lei walks 30 km in 6 hours. How far will he walk in 11 hours, walking at the same speed?
5. Emily drives a distance of 20 km at an average speed of 75 km/h, whilst Jacob drives a distance of 38 km at an average of 75 km/h. Who completes their trip in the shorter time?

6. Ethan lives 2 km from a quarry. There is a detonation (controlled explosion) at the quarry. If sound travels at approximately 340 metres/seconds, how many seconds pass before Ethan hears the detonation at his home?
7. Kiri has $30 and saves $ 10 per week. Joss has $300 but spends $10 per week.
   a. If Kiri keeps saving at the same rate and Joss keeps spending at the same rate, after how many weeks will they have the same amount of money?
   b. If Kiri saves $5 per week and Joss continues to spend $10 per week, after how many weeks will they have the same amount of money?
8. John is sitting a test with 24 questions to answer in 2 hours. He does a practice test of 17 questions which is timed at 1 hour 20 minutes. If John expects to work at the same rate in the actual test, can he complete it in time? If the answer is 'Yes', how many minutes will he have for checking the answers? If the answer is 'No', how many questions will John not complete?