

**Percentages**

**Percentages and Fractions**

- The word 'per cent' means 'per hundred'. The symbol % used to mean per cent (or 'out of a hundred').

**Converting a fraction to a percentage**

- When a fraction is already over 100, convert it directly into a percentage.  
- Multiply the fraction by 100% which is the perfect percentage over 1 then do the law of multiplication. [numerators multiply by numerators, denominators multiply by denominators]

**Example:**

i. Change  $\frac{9}{100}$  to a percentage.

This fraction is already over 100, therefore it can be directly written as a percentage.

$$\frac{9}{100} = 9\%$$

ii. Change  $\frac{2}{5}$  to a percentage.

$$\frac{2}{5} \times \frac{100\%}{1} = \frac{2 \times 100}{5 \times 1} \quad \text{[multiply } \frac{2}{5} \text{ by the perfect percentage which is}$$

100% over 1]

$$\frac{200}{5} = 40\% \quad \text{[simplify to get 40 %]}$$

iii. Change  $\frac{5}{6}$  to a percentage.

$$\frac{5}{6} \times \frac{100}{1} = \frac{5 \times 100}{6 \times 1} \quad \text{[multiply 5 by 100, 6 by 1]}$$

$$\frac{500}{6} = 83.33\% \quad \text{[500 divide by 6 to get 83.33 %]}$$

**Converting a percentage to a fraction**

- Put the percentage over 100 and simplify the fraction.

**Example:**

i. Convert 155% to its simplest fraction.

$$155\% = \frac{155}{100} \quad [\div 100]$$

$$= \frac{31}{20} \quad [\div 5]$$

$$= 1 \frac{11}{20}$$

ii. Convert  $12\frac{1}{2}\%$  to its simplest fraction.

$$12\frac{1}{2}\% = \frac{25}{2}\% \quad \text{[change into improper fraction]}$$

$$= \frac{25}{200} \quad [2 \times 100 = 200]$$

$$= \frac{1}{8} \quad \text{[simplify to its simplest form by } \div 25]$$

**Percentages and decimals**

- Decimals convert easily to percentages. Simplify the decimal by 100%.

**Example:**

Convert each of the following decimals to a percentage.

i. 0.43

$$0.43 = \frac{43}{100} \quad \text{[first change decimal into fraction]}$$

$$\frac{43}{100} \times \frac{100}{1} = \frac{43 \times 100}{100 \times 1} \quad \text{[multiply by 100 over 1]}$$

$$\frac{4300}{100} = 43\% \quad \text{[divided by 100 to get 43 %]}$$

ii. 0.6

$$0.6 = \frac{6}{10} \text{ [change decimal into fraction]}$$

$$\frac{6}{10} \times \frac{100}{1} = \frac{6 \times 100}{10 \times 1} \text{ [multiply by 100 over 1]}$$

$$\frac{600}{10} = 60\% \text{ [divide by 10 to get 60\%]}$$

Percentages convert easily to decimals. Divide the percentage by 100.

**Example:**

i. Convert 26% to a decimal

$$26\% = \frac{26}{100} \text{ [change into fraction first]}$$

$$26 \div 100 = 0.26$$

Therefore 26% is 0.26

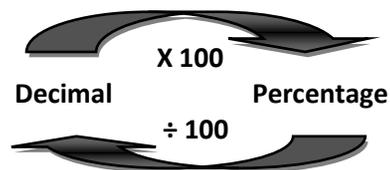
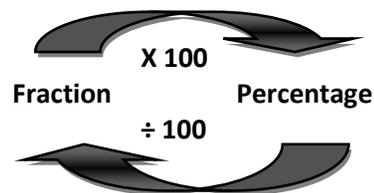
ii. Convert 7% to a decimal

$$7\% = \frac{7}{100} \text{ [change into fraction]}$$

$$7 \div 100 = 0.07$$

Therefore 7% is 0.07

**Notes:**



**Activity 7.1**

1. Write each of the following percentages as a fraction and simplify where possible

- |        |        |        |        |         |
|--------|--------|--------|--------|---------|
| a. 50% | b. 25% | c. 75% | d. 20% | e. 20%  |
| f. 15% | g. 44% | h. 65% | i. 5%  | j. 150% |

2. Write the following fractions as percentages.

- |                  |                   |                    |                  |                   |
|------------------|-------------------|--------------------|------------------|-------------------|
| a. $\frac{1}{8}$ | b. $\frac{1}{6}$  | c. $\frac{2}{3}$   | d. $\frac{5}{8}$ | e. $\frac{5}{12}$ |
| f. $\frac{2}{9}$ | g. $1\frac{1}{2}$ | h. $3\frac{2}{15}$ |                  |                   |

3. Convert the following decimals to percentages.

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| a. 0.47 | b. 0.34 | c. 0.26 | d. 0.98 | e. 0.05 |
| f. 0.04 | g. 1.76 | h. 1.09 | i. 2.34 |         |

4. Convert the following percentages to decimals.

- |          |          |        |         |          |
|----------|----------|--------|---------|----------|
| a. 57%   | b. 28%   | c. 89% | d. 90%  | e. 25.5% |
| f. 46.8% | g. 0.04% | h. 1%  | i. 126% |          |

**Finding a percentage of a quantity**

- For finding a fraction or decimal of a quantity:

- Convert the percentage to a fraction or a decimal.
- Multiply the quantity by this fraction or decimal.

**Example:**

i. Find 17% of \$43

$$17\% = \frac{17}{100} \text{ [change into fraction then divide]}$$

$$17 \div 100 = 0.17 \text{ [divide]}$$

$$0.17 \times \$43 = \$7.31 \text{ [multiply by the quantity given]}$$

ii. Find 123% of 52 kg

$$123\% = \frac{123}{100} \quad \text{[change into fraction]}$$

$$123 \div 100 = 1.23 \quad \text{[divide]}$$

$$1.23 \times 52 = \mathbf{63.96\text{kg}} \quad \text{[multiply by the quantity given]}$$

### Activity 7.2

1. Find 10% of the following amounts.

- |         |          |            |            |
|---------|----------|------------|------------|
| a. \$50 | b. 78 kg | c. \$34    | d. \$900   |
| e. 5 m  | f. 134 L | g. \$78.50 | h. \$16.20 |

2. Find 20% of the following amounts.

- |         |           |          |           |
|---------|-----------|----------|-----------|
| a. \$60 | b. 10 min | c. 25 cm | d. \$36   |
| e. \$65 | f. 120 mL | g. 2.4 g | h. \$3.90 |

3. Find the following percentages.

- |                  |                              |                              |
|------------------|------------------------------|------------------------------|
| a. 25% of 56 cm  | b. 75% of \$40               | c. $33\frac{1}{3}\%$ of \$90 |
| d. 12.5% of 88 L | e. 45% of 80 cm <sup>2</sup> | f. 15% of 120                |
| g. 1% of \$46    | h. 2% of \$46                | i. $66\frac{2}{3}\%$ of 15m  |

### Expressing one quantity as a percentage of another

- Expressing one quantity as a percentage of another means comparing two amounts that are in the same units.

i. Express the first quantity as a fraction of the second quantity by putting the first quantity over the second quantity.

ii. Change the fraction to a percentage by multiplying by 100.

### Example:

i. Express 2 cm as a percentage of 25 cm.

$$2 \text{ cm is } \frac{2}{25} \text{ of } 25 \text{ cm} \quad \text{[writing 2 cm as fraction of 25 cm]}$$

$$\frac{2}{25} \times 100\% = 8\% \quad \text{[changing fraction to percentage]}$$

**2 cm is 8% of 25cm.**

ii. Express 40 c as a percentage of \$2.

**40 c and \$2 are different units. Change \$2 to 200 c.**

$$40 \text{ is } \frac{40}{200} \text{ of } \$2 \quad \text{[writing 40 as fraction of 200]}$$

$$\frac{1}{5} \times 100\% = 20\% \quad \text{[simplifying } \frac{40}{200} \text{ to } \frac{1}{5}; \text{ changing fraction to percentage]}$$

**40 c is 20% of \$2.**

### Activity 7.3

1. Express the first quantity as a percentage of the second amount.

- |               |               |               |
|---------------|---------------|---------------|
| a. 6 and 24   | b. 17 and 50  | c. 23 and 25  |
| d. 11 and 20  | e. 9 and 10   | f. 45 and 60  |
| g. 16 and 128 | h. 60 and 180 | i. 12 and 480 |

2. Express the first quantity as a percentage of the second amount.

- |                  |                    |                   |
|------------------|--------------------|-------------------|
| a. 35 cm and 2 m | b. 30 c and \$2.40 | c. 36 cm and 1 m  |
| d. 8 mm and 5 cm | e. 160 cm and 1 km | f. 400 g and 1 kg |

**Percentage change**

- When a quantity changes from one amount to another, this change can be **expressed as a percentage of the original quantity**.

- i. Find the change in the amount.
- ii. Divide the change in the amount by the ORIGINAL amount. (This can form a decimal or a fraction.)
- iii. Change the decimal or fraction to a percentage.

**Example:**

i. The membership numbers at Racket Tennis Club Increase from 240 members to 300 members. What is the percentage change in membership?

**Step 1: Change in amount**

$$300 - 240 = 60$$

**Step 2: Required fraction or divide change in amount by the Original amount**

$$\frac{60}{240} = \frac{1}{4}$$

**Step 3: Change into percentage**

$$\frac{1}{4} \times \frac{100}{1} = \frac{1 \times 100}{4 \times 1}$$

$$\frac{100}{4} = \mathbf{25\%}$$

**Activity 7.4**

1. Calculate the following percentage changes. Round answers to 1 dp where necessary.

- |                   |                   |                       |
|-------------------|-------------------|-----------------------|
| a. 20°C to 30 °C  | b. \$70 to \$85   | c. \$35 to \$40       |
| d. 90 g to 80 g   | e. \$45 to \$30   | f. 5 min to 7 min     |
| g. 38 kg to 56 kg | h. 980 L to 450 L | i. \$12.50 to \$26.70 |

2. A camera shop reduced the price of a camera from \$680 to \$570.

a. What is the actual reduction?

b. Calculate the percentage reduction.

3. The temperature has risen from 14 °C to 20 °C. What is the percentage change in temperature? Round answer to 1 dp.

**Increasing or decreasing by a percentage**

- 'Bus fares increase by 5%', 'Rates increase by 12%', 'Workforce reduces by 5%'. Most quantities in life change.

**- Calculating the new quantity after an increase or decrease:**

- i. Find the actual amount of increase or decrease of the quantity.
- ii. For an increase add the amount to the original quantity.
- iii. For a decrease subtract the amount from the original quantity.

**Example:**

i. Calculate the new price when \$56 is **increased** by 20%

**Step 1: Find 20% of \$56**

$$20\% \div 100\% = 0.2 \quad \text{[divide 20% by 100% (perfect percentage)]}$$

$$0.2 \times \$56 = \$11.20 \quad \text{[multiply by the given quantity]}$$

$$\text{So increase is } \$11.20 \quad \text{[amount increased]}$$

**Step 2: Add the increased amount to the original quantity**

$$\$56 + \$11.20 = \$67.20$$

**New Price = \$67.20**

ii. The average daily number of passengers that take the train in May is 280. In June this **drops** by 15%. How many passengers take the train daily on average in June?

**Step 1: Find 15% of 280**

$$15\% \div 100\% = 0.15 \quad [\text{divide by } 100\%]$$

$$0.15 \times 280 = 42 \quad [\text{multiply by the given quantity}]$$

$$\text{So decrease is } 42 \quad [\text{number decreased}]$$

**Step 2: Subtract the amount from the original quantity**

$$280 - 42 = 238$$

**Number of passengers in June = 238 passengers**

### Activity 7.5

1. Increasing the following amounts by the given percentage.

- |                |                              |                 |
|----------------|------------------------------|-----------------|
| a. \$50 by 10% | b. \$60 by 20%               | c. \$70 by 25%  |
| d. \$90 by 75% | e. \$30 by $33\frac{1}{3}\%$ | f. \$45 by 10%  |
| g. \$50 by 5%  | h. \$100 by 12.5%            | i. \$200 by 60% |

2. Decrease the following amounts by the given percentage.

- |                |                              |                 |
|----------------|------------------------------|-----------------|
| a. \$50 by 10% | b. \$60 by 20%               | c. \$70 by 25%  |
| d. \$90 by 75% | e. \$30 by $33\frac{1}{3}\%$ | f. \$45 by 10%  |
| g. \$50 by 5%  | h. \$100 by 12.5%            | i. \$200 by 60% |

3. Calculate the new quantities after the following increase or decrease.

- |                             |                              |
|-----------------------------|------------------------------|
| a. Increase 56 m by 40%     | b. Decrease 120 cm by 35%    |
| c. Increase \$40 000 by 14% | d. Decrease \$56 000 by 16%  |
| e. Increase 78 kg by 6%     | f. Decrease 980 tonnes by 3% |
| g. Increase 78 000 by 1%    | h. Decrease 1.3 by 17%       |

4. Georgia gets paid \$750 a week. She gets a 12% rise. What is her new weekly salary?

### Applications of percentages

- Business applications use the following vocabulary.

- *retailer* – a shopkeeper (faifakatau)
- *cost price* – the price at which a retailer **buys** (fakatau mai) stock
- *selling price* – the price at which a retailer **sells** (fakatau atu) stock
- *profit* – the amount added to a cost price to arrive at a selling price (or mark-up / tupu)
- *percentage profit* – profit expressed as a % of the cost price
 
$$\% \text{ profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100\%$$
- *discount* – amount subtracted from a selling price; discounts are often expressed as a percentage of the selling price.

**Example:**

i. A retailer buys a cycle for \$150 and sells it for \$300. What is his percentage profit?

Selling price = \$300, Cost price = \$150

Profit = selling price – cost price

$$\begin{aligned} \text{Profit} &= 300 - 150 \\ &= \$150 \end{aligned}$$

$$\begin{aligned} \% \text{ profit} &= \frac{\text{Profit}}{\text{Cost Price}} \times 100\% \\ &= \frac{150}{300} \times 100\% \\ &= 50\% \end{aligned}$$

**The percentage profit is 50%**

ii. A furniture shop has some old stock of coffee tables, which normally sell for \$200 each. If the shopkeeper sells them for \$160 each, what is the percentage discount?

$$\begin{aligned} \text{Discount} &= \$200 - \$160 \\ &= \$40 \end{aligned}$$

$$\begin{aligned} \% \text{ discount} &= \frac{\text{Discount Price}}{\text{Selling Price}} \times 100\% \\ &= \frac{40}{200} \times 100 \\ &= 20\% \end{aligned}$$

**The percentage discount is 20%**

**Interest**

- Interest is money paid for the use of money (the principal). Interest is expressed as a percentage rate.

i. **Simple interest** is worked out using the formula:

$$\text{Simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

**Example**

The simple interest on \$400 at 3% for 2 years is:

$$\begin{aligned} \text{Simple interest} &= 400 \times 0.03 \times 2 && [3\% / 100\% = 0.03] \\ &= \$24 \end{aligned}$$

ii. **Compound interest** includes the interest with the principal at the end of each **period** (time over which the interest is calculated).

**Example**

\$400 is invested at 3% compound interest per year for 2 years.

At end of year 1, investment is worth:

$$400 \times 1.03 = \$412 \quad [103\% \text{ of previous value}]$$

At end of year 2, investment is worth:

$$412 \times 1.03 = \$424.36$$

Total interest = \$424.36 – \$400

$$= \underline{\underline{\$24.36}}$$

**Note:** A single calculation of  $400 \times 1.03 \times 1.03$  (or  $400 \times 1.03^2$ ) will give the value of the investment after 2 years. (The value of the investment after 5 years would be  $400 \times 1.03^5 = \$463.71$ )

**Activity 7.6**

1. Fane buys in a pair of golf shoes for \$189 and sells them to a customer for \$289.50. Find her percentage profit on the gold shoes.
2. A hedge-trimmer is on sale at a cash price of \$460. Semisi pays a deposit of \$46, plus \$15 per month for 3 years. After 3 years, how much has Semisi paid for the trimmer?
3. Meleliku is buying a new bed on HP. The cash price is \$2 100, and she pays a deposit of 10%. The money she owes, which includes an interest rate of 20%, is paid off with 24 equal payments. How much is each payment?
4. Simote works in a hospital and is paid \$24.50 per hour. For each hour he works over 40 hours per week, he is paid an overtime rate of time and a half. If Andy pays tax at 30%, what is his take-home pay for a week in which he does 48 hours work?
5. Samiu puts a make-up of 60% on a softball bat, which he then sells for \$152. What is the cost price?

**Purchase options** - Different methods of payment are available for buying goods

**Bank credit card** - The goods are available immediately for use. Money owing on the credit card is paid at a later date.

**Hire purchase (HP)** - Pay cash deposit (e.g.20% of the price) then pay the rest in regular instalments. The retailer arranges with a finance company to lend the extra money. Extra money is paid in interest.

**Cash payment** - Save the money and buy with a cash payment. This is the cost of the goods with no interest added. This could take some time.

**Bank loan** - The bank lends the cash required. Bank is paid back at an agreed sum per week. The interest charged by the bank is usually lower than the interest charged by a finance company