

Decimals

Decimal place value

- The decimal system is based on the number 10.
- The **place-value 'house'** can now be extended to show fractional place value in decimal numbers.
- A decimal point is placed **after the ones column** in a table that now has columns for tenths, hundredths and thousandths.
- The decimal point **separates** the **whole-number part** from the **fraction part**.

Hundreds	Tens	Ones	.	Tenths	Hundredths	thousandths
		0	.	1		

- One tenth is written as 0.1 or $\frac{1}{10}$ (one in the tenths column).
- One hundredth is written as 0.01 or $\frac{1}{100}$ (one in the hundredths column).
- One thousandth is written as 0.001 or $\frac{1}{1000}$ (one in the thousandths column)

Example:

Show the place values for the digits in the number 45.297
- Place the number in a place-value table.

Hundreds	Tens	Ones	.	Tenths	Hundredths	thousandths
	4	5	.	2	9	7

The value for each digit can be read from the table.

- 4 tens = 40
- 5 ones = 5
- 2 tenths = 0.2
- 9 hundredths = 0.09
- 7 thousandths = 0.007

In **expanded form**: $4 \times 10 + 5 \times 1 + 2 \times 0.1 + 9 \times 0.01 + 7 \times 0.001$

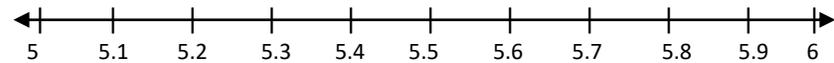
In words, 45.297 is **'forty-five point two nine seven'**.

Decimal number line

- One way of comparing the size of decimals is to place the decimal numbers on a number line.

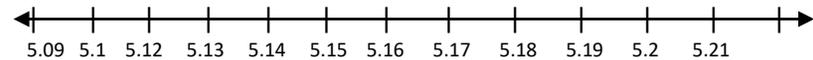
i. Tenths number line

- The line is divided into **tenths**. Each interval is of width **0.1 (1 tenth)**, e.g. the section between 5 and 6 is divided into tenths in the number line below.



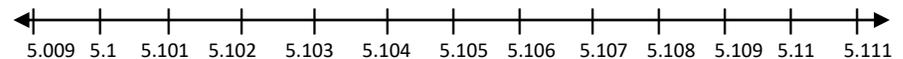
ii. Hundredths number line

- Each interval of width **one tenth** can be divided into ten equal sections, each of width 0.01 (**1 hundredth**) e.g. the section between 5.1 and 5.2 is divided into hundredths.



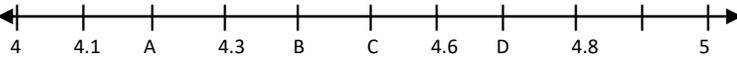
iii. Thousandths number line

- Each interval of width **one hundredth** can be divided into ten equal sections, each of width 0.001 (1 thousandth). e.g. the section between 5.1 and 5.11 is divided into thousandths.



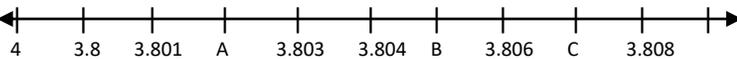
Example:

1. Name the marked decimals.



A = 4.2 [A is between 4.1 and 4.3, therefore it is 4.2]
 B = 4.4 [B and C are between 4.3 and 4.6, therefore B is 4.4 and C is 4.5]

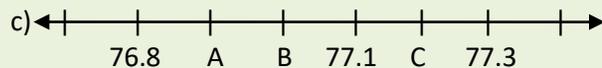
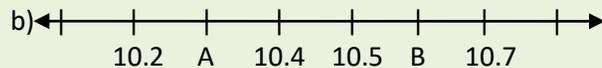
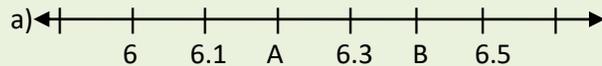
2. Name the marked divisions.



A = 3.802 [A is between 3.801 and 3.803, therefore it is 3.802]
 B = 3.805 [B is between 3.804 and 3.806, therefore it is 3.805]
 C = 3.807 [B is between 3.806 and 3.808, therefore it is 3.807]

Activity 6.1

1. Fill in the missing decimals on the following number lines.



2. Write the following in words.

- a. 54.8 b. 102.67 c. 3.089 d. 12.572

3. Write the following decimals in expanded form.

- a. 34.8 b. 23.67 c. 5.678 d. 3.806

Decimal fractions

- Each place value is 10 times greater than the place value on the right.

Example:

i. Write **0.45** as a fraction.

- This number has **4 tenths** and **5 hundredths**.

$\frac{0.45}{100} = \frac{45}{100}$ [place 1 under the decimal point then replace the rest of the numbers by zeros]

$\frac{45}{100} = \frac{9}{20}$ [can simplify to its simplest form]

ii. Write 0.405 as a fraction.

$\frac{0.405}{1000} = \frac{405}{1000}$ [place 1 under the decimal point then replace the rest of the numbers by zeros]

$\frac{405}{1000} = \frac{81}{200}$ [can simplify to its simplest form]

iii. Write $\frac{3246}{100}$ as a decimal.

- Hundredths means there will be 2 decimal places

- Move the decimal point twice (2 times) to the left of the number

3246 [now the decimal point is between 2 and 4]

32.46 [now the answer is 32.46]

Recurring decimals

- The fraction $\frac{1}{3}$ when put into a calculator gives a **recurring decimal**. 0.3333333... the digits repeat infinitely.
- A **recurring decimal** is written with a **dot or line over the recurring part** of the decimal. So 0.333333... is written as 0. $\overline{3}$.

The table below gives some common recurring decimals.

Fraction	Decimal	Recurring decimal
$\frac{1}{3}$	0.33333	0. $\overline{3}$
$\frac{1}{9}$	0.11111	0. $\overline{1}$
$\frac{4}{9}$	0.44444	0. $\overline{4}$

Activity 6.2

1. Write as a fraction.

- a. 0.3 b. 0.7 c. 0.78 d. 6.5 e. 2.45
f. 3.79 g. 0.6 h. 2.467 i. 5.264 j. 80264

2. Write in decimal form.

- a. $\frac{43}{10}$ b. $\frac{523}{10}$ c. $\frac{254}{100}$ d. $\frac{2678}{100}$ e. $\frac{968}{100}$
f. $\frac{7853}{1000}$ g. $\frac{458767}{10}$ h. $\frac{6782}{100}$ i. $\frac{5}{100}$ j. $\frac{27}{100}$

Multiplying decimals by powers of ten

- Multiplying a digit by 10, moves the decimal point one place to the right of the digit.
- Multiplying a digit by the powers of ten, move the decimal point according to the number of zeros in the powers of ten to the right of the digit.

Example:

i. 2.456×10

2 | . | 4 | 5 | 6 |



Move the decimal point one to the right, now the decimal point is between 4 and 5

So $2.456 \times 10 = 24.56$

ii. 6.927×100

6 | . | 9 | 2 | 7 |



Since there are 2 zeros in 100, so move the decimal point 2 times. Now the decimal point is between 2 and 7,

So $6.927 \times 100 = 692.7$

Dividing decimals by powers of ten

- Dividing a digit by 10, move the decimal point one place to the left of the digit.
- Dividing a digit by the powers of ten, move the decimal point according to the number of zeros in the powers of ten to the left of the digit.

Example:

i. $25.34 \div 10$

2 | 5 | . | 3 | 4 |



- Move the decimal point one time to the left, now the decimal point is between 2 and 5

So $25.34 \div 10 = 2.534$

ii. $324.5 \div 100$

3 | 2 | 4 | . | 5 |



- Since there are 2 zeros in 100, so move the decimal point 2 times. Now the decimal point is between 3 and 2,

So $324.5 \div 100 = 3.245$

Activity 6.3

1. Evaluate.

- | | | |
|------------------------|-----------------------|------------------------|
| a. 45×10 | b. 38.7×100 | c. $0.87 \times 1,000$ |
| d. 37×100 | e. 38.008×10 | f. $0.98 \times 1,000$ |
| g. 745.01×100 | h. 0.042×100 | i. $6.8 \times 10,000$ |

2. Evaluate the following.

- | | | |
|----------------------|---------------------|-----------------------|
| a. $32 \div 10$ | b. $786 \div 100$ | c. $3956 \div 1,000$ |
| d. $786 \div 10,000$ | e. $376.9 \div 100$ | f. $275.9 \div 1,000$ |
| g. $2.67 \div 10$ | h. $0.67 \div 100$ | i. $3.709 \div 1,000$ |

Adding and subtracting decimals

- The same strategies used for adding and subtracting whole numbers can be used for adding and subtracting decimals.

Example:

i. $14.5 + 10.24$

Tens	Ones	.	Tenths	hundredths
1	4	.	5	
+ 1	0	.	2	4
2	4	.	7	4

ii. $86.54 - 4.07$

Tens	Ones	.	Tenths	Hundredths
8	6	.	5	4
-	4	.	0	7
8	2	.	4	7

Activity 6.4

1. Calculate the following. Show your working.

- | | | |
|------------------------|-------------------------|--------------------------|
| a. $4.3 + 8.7$ | b. $3.49 + 2.51$ | c. $23.43 + 8.5$ |
| d. $3.8 + 12.2$ | e. $14.4 + 2.6$ | f. $4.88 + 3.12$ |
| g. $15.9 - 6.4$ | h. $27.57 - 3.23$ | i. $4 - 2.05$ |
| j. $4.7 + 3.48 + 3.3$ | k. $4.6 + 8.9 - 1.7$ | l. $271 + 4.63 + 5.86$ |
| m. $7.4 + 8.9 + 11.64$ | n. $28.7 + 17.3 - 14.3$ | f. $25.01 + 4.89 + 3.99$ |

2. Leni buys three shirts costing \$42.72, \$53.12 and \$98.53.

- How much did he spend altogether?
- How much change did he get from \$200?

3. Asikati is practicing the high jump. On the first jump she clears 12.56 cm, on the second jump she clears 127.8 cm.

- What is the difference between the two jumps?
- How many more centimetres above her second jump does Asikati need to clear to reach her target height of 133 cm?

Multiplying decimals

Simple decimal multiplication

$$300 \times 4 = 3 \times 4 \times 100 = 1\ 200$$

$$30 \times 4 = 3 \times 4 \times 10 = 120$$

$$3 \times 4 = 3 \times 4 \times 1 = 12$$

$$0.3 \times 4 = 3 \times 4 \times 0.1 = 1.2$$

$$0.03 \times 4 = 3 \times 4 \times 0.01 = 0.12$$

The number of digits after the decimal point (decimal places) in your answer should equal the total number of decimal places in the equation

Example:

i. 6×0.9

$$6 \times 0.\underline{9} = 6 \times 9 \times 0.1$$

$$= \underline{5.4}$$

[1 digit after the decimal point in the question, therefore 1 digit after the decimal point in the answer]

ii. 4×0.08

$$4 \times 0.\underline{08} = 4 \times 8 \times 0.01$$

$$= \underline{0.32}$$

[2 digit after the decimal point in the question, therefore 2 digit after the decimal point in the answer]

iii. 0.23×0.5

$$0.\underline{23} \times 0.\underline{5} = 23 \times 5 \times 0.001$$

$$= \underline{0.115}$$

[3 digit after the decimal point in the question, therefore 3 digit after the decimal point in the answer]

Dividing decimals

- The division strategies for whole numbers can then be used.

Example

i. $8 \div 0.4$

-This means how many 0.4 fit into 8.

- Multiply both sides by 10, so $8 \times 10 = 80$ and $0.4 \times 10 = 4$

$80 \div 4 = 20$, So $80 \div 0.4 = 20$

ii. $6.5 \div 0.5$

-Multiply both by 10, so $6.5 \times 10 = 65$ and $0.5 \times 10 = 5$

$65 \div 5 = 13$, So $6.5 \div 0.5 = 13$

iii. $9.8 \div 1.4$

Multiply both sides by 10, so $9.8 \times 10 = 98$ and $1.4 \times 10 = 14$.

$98 \div 14 = 7$, So $9.8 \div 1.4 = 7$

Activity 6.5

1. Evaluate the following.

a. 3×0.7

b. 4×0.2

c. 4×0.09

d. 0.02×8

e. 11×0.4

f. 1.2×7

g. 2.4×5

h. 9×1.2

h. 0.3×0.4

2. Calculate without using a calculator.

a. $8 \div 0.2$

b. $6 \div 0.4$

c. $10 \div 0.4$

d. $12 \div 0.6$

e. $24 \div 0.8$

f. $16 \div 0.8$

g. $35 \div 0.7$

h. $40 \div 0.5$

i. $10 \div 0.02$

Decimal rounding

- Rounding gives fewer decimal places and is a more practical way to write an answer.

- 'Decimal place' can be written as **dp**

Way of doing decimal rounding

Step 1: Go to the digit in the decimal place given

Step 2: See if the digit at the most right of it is less than or greater than 5

Step 3: If it's greater than or equal 5, add 1 to the digit in the given decimal place,

Else, if it's less than 5, number stays the same but eliminate (cancel out) all digits at the right side of the digit in the given dp.

Example:

i. Round 3.454 to 2 decimal places (2 dp)

$$\begin{array}{ccccccc}
 3 & . & & 4 & & 5 & & 4 \\
 & & & \downarrow & & \downarrow & & \\
 & & & 1 \text{ dp} & & 2 \text{ dp} & &
 \end{array}$$

5 is in 2 dp, therefore 4 is at the most right of 5 which is less than 5, number stays the same but cancel out all the digits at the right side of 5.

So **3.454 = 3.45 in 2 dp**

ii. Round 4.89 to 1 dp

$$\begin{array}{ccccccc}
 4 & . & & 8 & & 9 & &
 \end{array}$$

8 is in 1 dp, so digit at the most right of 8 is 9, which is greater than 5, so add 1 to 8 to become 9.

So **4.89 = 4.9 in 1 dp**

Activity 6.6

1. Round the following numbers to 1 dp.

- a. 3.67 b. 4.73 c. 12.87 d. 24.03
e. 45.08 f. 1.54 g. 7.55 h. 8.89

2. Round the following numbers to 2 dp.

- a. 5.724 b. 4.879 c. 23.444 d. 0.079
e. 0.195 f. 23.454 g. 6.999 h. 99.999

3. Calculate the following then round answers to 1 dp.

- a. $4 \div 9$ b. $2 \div 3$ c. 3.1×2.4 d. 9.7×4.8
e. 0.6×7.3 f. 16.5×14.9

4. At the Beijing Olympics in 2008, Usain Bolt ran the 100m in 9.69 seconds. What is this time to 1 decimal place?

5. Usain Bolt set a new world record in 2009 for the 100 m in a time of 9.58 seconds. What is this time to 1 decimal place?