

Logics

Sets – A set is a **collection of objects**.

- Each **object** is called a **member or element** of the set.
- Sets are often **named** with **capital letters**.

$A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$

- When a set has **no elements**, it is the **empty set**. You write $\{\}$ or \emptyset to indicate the empty set.

$A = \{\}$

Example:

i. Let F be the set of factors of 30, find F?

$F = \{1, 2, 3, 5, 6, 10, 15, 30\}$

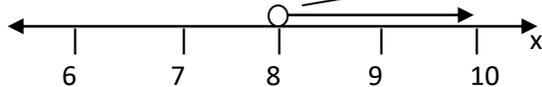
ii. Write the following sets as inequalities and in interval notation:

a) G, the set of real numbers **greater than 8**

means 8 does not include

set notation: $G = \{x : x > 8\}$ or using **interval notation:** $x \in (8, \infty)$

- This set can also be shown on a number line, where an **open circle** means that the number is **not included**

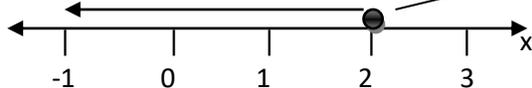


b) H, the set of real numbers less **than or equal to 2**

means 2 is included

set notation: $H = \{x : x \leq 2\}$ or using **interval notation:** $x \in (-\infty, 2]$

- This set can also be shown on a number line, where a **closed circle** means that the number is **included**



Activity 9.1

1. List the elements in the following sets:

- a. $H = \{\text{the factors of } 48\}$
- b. $K = \{\text{the integers greater than } -4 \text{ and less than } 7\}$
- c. $L = \{\text{the natural numbers greater than } -3 \text{ and less than } 5\}$
- d. $P = \{\text{the negative integers greater than } -5\}$

2. Display the following sets on a number line: (x belongs to the set of real numbers)

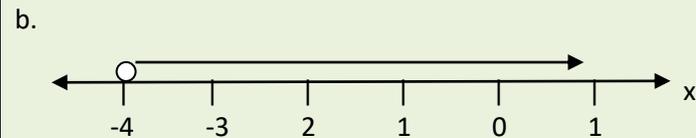
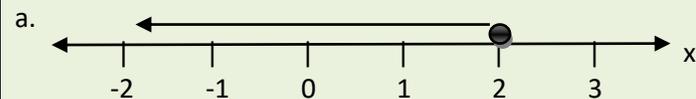
- a. $\{x : x \geq 7\}$
- b. $\{x : x < 4\}$
- c. $\{x : x \geq -3\}$
- d. $\{x : x \geq -1\}$

3. Display the following on a number line

- a. $(-4, 2)$
- b. $(-\infty, 6)$
- c. $[-1, \infty)$
- d. $(5, 8)$

4. Describe the following using:

- i. set notation
- ii. interval notation



Subsets - When all the elements of a set are **also elements of another set**.

Example:

The first set is a subset of the other set

$\{2, 4\} \subset \{1, 2, 3, 4\}$

(\subset is the subset symbol)

Find all the subsets of each set.

- i. $\{1, 4\} = \{1\}, \{4\}$
- ii. $\{m\} = \{m\}$
- iii. $\{a, b, c\} = \{a\}, \{b\}, \{c\}, \{a,b\}, \{a, c\}, \{c, b\}$

[Remember that every set of itself and that the empty set is a subset of every set].

Union of Sets - You find the **union of two sets** by creating a new set that contains all of the elements from the two sets.

Example:

$$J = \{1, 3, 5, 7\} \quad L = \{2, 4, 6, 8\}$$

$$J \cup L = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

(\cup is the union symbol)

Find the union of each pair of sets

- i. $\{1, 2\} \cup \{9, 10\} = \{1, 2, 9, 10\}$
- ii. $\{m, a, t, h\} \cup \{m, a, p\} = \{a, h, m, p, t\}$
- iii. $\{_, \$, _, \%, _, \infty\} \cup \{\infty, \%, \$, \#\} = \{\#, \$, \%, _, \infty\}$

Intersection of sets - You find the **intersection of two sets** by creating a new set that contains all of the elements that are common to both sets.

Example:

$$A = \{8, 12, 16, 20\} \quad B = \{4, 8, 12\}$$

$$A \cap B = \{8, 12\}$$

(\cap is the intersection symbol.)

[If the sets have no elements in common, the intersection is the empty set \emptyset].

Find the intersection of each pair of sets

- i. $\{9\} \cap \{9, 18\} = \{9\}$
- ii. $\{a, c, t\} \cap \{b, d, u\} = \emptyset$
- iii. $\{_, \$, _, \%, _, \infty\} \cap \{\infty, \%, \$, \#\} = \{\$, \%, \infty\}$

Activity 9.2

1. State whether each statement is **true** or **false**.

- a. $\{1, 2, 3\} \subset \{\text{counting numbers}\}$
- b. $\{1, 2, 3\} \subset \{1, 2\}$
- c. $\{1, 2, 3\} \subset \{\text{even numbers}\}$
- d. $\{1, 2\} \subset \{1, 2, 3\}$

2. Find the union of each pair of sets.

- a. $\{2, 3\} \cup \{4, 5\}$
- b. $\{x, y\} \cup \{y, z\}$
- c. $\{r, o, y, a, l\} \cup \{m, o, a, t\}$
- d. $\{2, 5, 7, 10\} \cup \{2, 7\}$

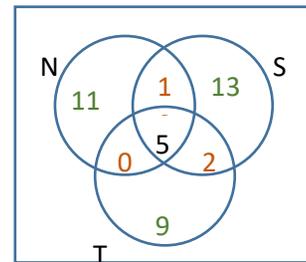
3. Find the intersection of each pair of sets.

- a. $\{1, 3, 5, 7\} \cap \{6, 7, 8\}$
- b. $\{r, o, y, a, l\} \cap \{b, e, t\}$
- c. $\{r, o, y, a, l\} \cap \{m, o, a, t\}$
- d. $\{2, 5, 7, 10\} \cap \{4, 5\}$

Venn Diagrams

- A Venn diagram shows how the elements of two or more sets are related. When the circles in a Venn diagram overlap, the overlapping part contains the elements that are common to both sets.

- When evaluation Venn diagrams, you have to look carefully to identify the overlapping parts to see which elements of the sets are in those parts.



These are sets from the Venn diagram above:

$$N = \{0, 1, 5, 11\}$$

$$S = \{1, 2, 5, 13\}$$

$$T = \{0, 2, 5, 9\}$$

$$N \cup S = \{0, 1, 2, 5, 11, 13\}$$

$$N \cup T = \{0, 1, 2, 5, 9, 11\}$$

$$S \cup T = \{0, 1, 2, 5, 9, 13\}$$

$$N \cup S \cup T = \{0, 1, 2, 5, 9, 11, 13\}$$

$$N \cap S = \{1, 5\}$$

$$N \cap T = \{0, 5\}$$

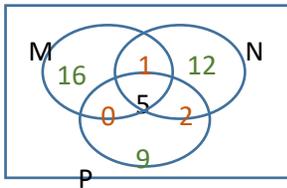
2. Draw a Venn diagram to help answer the following questions.

15 students stood in a queue at the school café. When they had bought what they wanted, they found that 9 had bought a chocolate bar, 8 had bought chippies and 5 had bought both. The rest had bought a drink only.

- How many had bought a drink?
- How many did not buy a chocolate bar?
- How many did not buy chippies?
- How many bought either just a chocolate bar or just a drink?

Activity 9.3

1. Use the Venn diagram at the below to list the following sets.



- List the elements of set M.
- List the elements of set N.
- Find P.
- Find $M \cup N$
- Find $N \cup P$
- Find $M \cup P$
- Find $M \cap N$
- Find $P \cap N$
- Find $M \cap N \cap P$