Strand: Algebra

Patterns and relationships

- The study of **patterns** is at the heart of mathematics, if you can spot the patterns, this will help you to recognize **relationships** in mathematics.

Sequences

- A sequence is an **ordered list** of numbers, letters or shapes.
- A number sequence is a **set of numbers** such as 2, 4, 6,8…….
- Each number in a sequence is called a **term, t.**
- The first term is 2 written as t1 = 2, the second term is 4, t2 = 4 and so on.

Example:

What is the next term for the following sequences?

i. 5, 8, 11, 14, ….

The sequence 5, 8, 11, 14…. Has the **pattern 'add 3'.**

The **next term** would be **14 + 3 = 17** (t5 = 17), therefore:

t1 = 5, t2 = 8, t3 = 11, t4 = 14, **t5 = 17,** …

ii. The first number of a sequence is 6. Continue by adding 5 each time. Find the first five terms of the sequence.

t1 = 6, [given]

t2 = 6 + 5 = 11	[t2 = t1 + 5]
t3 = 11 + 5 = 21	[t3 = t2 + 5]
t4 = 16 + 5 = 21	[t4 = t3 + 5]
t5 = 21 + 5 = 26	[t5 = t4 + 5]

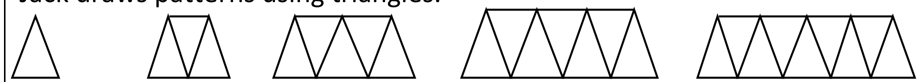
Therefore the **first five terms** of the **sequence** are 6, 11, 16, 21, 26

Patterns

- Patterns occur in nature and many relate to **mathematical shapes.**
- Number sequences can be generated from pattern of shapes. These are called **spatial patterns.**

Example:

Jack draws patterns using triangles.



pattern 1 pattern 2 pattern 3 pattern 4 pattern 5

Jack counts the number of triangles used in each pattern and puts the results in the table alongside.

Jack wants to know how many triangles will be used for the **sixth pattern** (pattern 6).

Pattern number	Number of triangles
1	1
2	3
3	5
4	7
5	9

Jack sees the **pattern rule** is **'add (+) 2 to each term'** in the number sequence. So the next term in the sequence will be **9 + 2 = 11.**

- So Jack will use **11 triangles** in the **6th pattern.**

Activity 10.1

1. Find the missing numbers in the sequences.

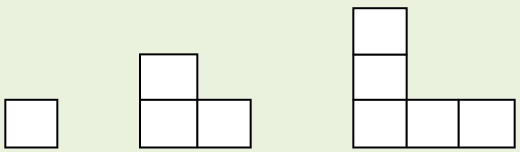
a. 2, 4, ____, 8, 10
b. 3, 6, 9, ____, 15
c. 80, 40, 20, ____, 5
d. 1, 2, 4, 7, ____, 16
e. 10.5, 8.5, 6.5, ____, 2.5
f. -6, -3, ____, 3, 6

2. Find the next 3 terms in the sequences.

a. 5, 10, 15, ____, ____, ____
b. 60, 51, 42, ____, ____, ____
c. $\frac{1}{2}$, 1, 2, 4, ____, ____, ____
d. 5 000, 500, 50, ____, ____, ____
e. 4, 9, 16, 25, ____, ____, ____

3. Write down the first five terms of these sequences.
a. The first number is 6. Add 5 each time.
b. The first number is 32. Subtract 4 each time.
c. The first number is 2. So duble each time.
d. The first number is 120. Half each time.
e. The first term is 2. Square each time.

4a. Draw the next two figures in the spatial sequence.



b. Copy and complete the table below.

Pattern number (n)	Number of triangles (s)
1	1
2	3
3	5
4	
5	

Linear sequence

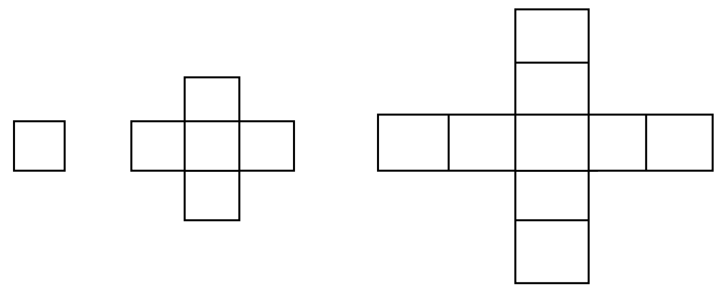
- A **linear sequence** has terms that **increase (+) or decrease (-)** by the **same amount.**
- The pattern will involve (or subtracting) the same number each time.
- An **iterative rule** tells you how to get from one term to the next.

Example:
i. Show the sequence 5, 7, 9, 11, 13, … is a linear sequence.
- Starting at 5, the **rule is 'add 2'**. The terms **increase** each time by **2**, so the sequence is linear.

ii. Show the sequence 3, 5, 8, 12, 17, … is not a linear sequence.
- The terms increase by 2, then 3, then 4, then 5, … The terms **do not increase** by the **same number** each time, so the sequence is **not linear.**

- The number sequence generated from a spatial sequence can have a linear pattern.

Example:
i. Find the number sequence for the spatial pattern **made of squares**, as shown below. Give the **iterative rule** for this sequence.



Pattern 1 Pattern 2 Pattern 3

Write down the rule for the sequence formed by counting squares.
Pattern 1 has 1 square
Pattern 2 has 5 squares
Pattern 3 has 9 squares
Pattern 4 might be 13 square
Pattern 5 might be 17 squares

Therefore the number sequence from this **spatial pattern is 1, 5, 9, 13, 17, …**

The **iterative rule** for this sequence is **'add 4',** starting at 1.

Activity 10.2

1. Find the next two numbers in the sequence. Explain why the sequence is linear or why it is not.

a. 25, 20, 15, 10, ___, ___

b. 2, 4, 8, 16, ___, ___

c. 3, 9, 27, 81, ___, ___

d. 4, 7, 10, 13, ___, ___

e. 100, 10, 1, ___, ___

f. 3.5, 4.7, 5.9, 7.1, ___, ___

2. Consider the patterns made of dots below. For each one:

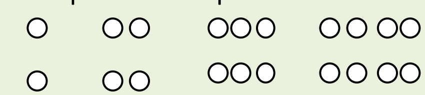
i. Draw the next two shapes in the sequence.

ii. Count the dots in each shape and write down the first five number terms of the sequence

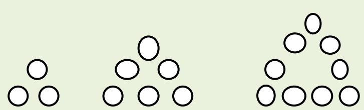
iii. Find the number of dots there would be in the 8th pattern

iv. Write, in words, the iterative rule used to find the number of dots for the next shape in the sequence.

a.



b.



General rule for a linear sequence

- When the first few terms of a sequence are given, an **iterative rule** such as 'add 2' or 'subtract 3' is useful for finding the next few terms.
- This type of rule is not so useful for finding terms further along the sequence, **e.g.** the 100th term.
-A general rule links the position of the term in the sequence with its value. - This rule may be stated in words.

Example:

The rule for a sequence is **'double the pattern number then add 3'.**

The 1st term is 2 x 1 + 3 = 5

The 2nd term is 2 x 2 + 3 = 7

The 3rd is 2 x 3 + 3 = 9

The 4th term is 2 x 4 + 3 = 11 and so on

This gives the linear sequence 5, 7, 9, 11, …

Rules are more often given algebraically.

- An **algebraic rule** is **made up** of at least **one variable (a letter).** By putting different term numbers in place of the variable, a number sequence can be generated.
- Once the general rule for a pattern is known, terms can be found by **substitution.**

Example:

i. The working for the first four terms of the sequence whose rule is **'double the pattern number then add 3'** is shown in the table below.
Find the algebraic rule.

- The numbers 2 and 3 are in each line of the working. The other number changes. This is the variable, which is the pattern number.
- Using **'n'** for the pattern number, the **final line** of the tables shows the **'general rule'.**

Pattern number (n)	Terms
1	$2 x 1 + 3 = 5$
2	$2 x 2 + 3 = 7$
3	$2 x 3 + 3 = 9$
4	$2 x 4 + 3 = 11$
n	$2 x n + 3$ or $2n + 3$

The rule for the pattern is $\mathbf{2n + 3}$

ii. Find the first 3 terms and the 30th term of the sequence whose rule is $3n + 2$, where **n** is the variable standing for the pattern number.

$t1 = 3 \times 1 + 2 = 5$ [replace n with 1 to get the 1st term]
$t2 = 3 \times 2 + 2 = 8$ [replace n with 2 to get the 2nd term]
$t3 = 3 \times 3 + 2 = 11$ [replace n with 3 to get the 3rd term]
$t30 = 3 \times 30 + 2 = 92$ [replace n with 30 to get the 30th term]

Therefore the number sequence is 5, 8, 11…. **And the 30th term is 92.**

Or, the working can be shown in a table.

Term (t)	$3 \times n + 2$ or $3n + 2$	Value
1	$3 \times 1 + 2$	5
2	$3 \times 2 + 2$	8
3	$3 \times 3 + 2$	11
30	$3 \times 30 + 2$	92

Activity 10.3

1a.Copy and complete the table below using the given rule $3 \times n + 2$.

Term (t)	$3 \times n + 2$	Value
1	$3 \times 1 + 2$	5
2	$3 \times 2 + 2$	8
3		
4		
5		
6		

b. Describe any patterns you can see in your results.
c. What is the value of the 20th term?

2. Copy tables and complete by finding the first 6 terms and the 30th term of the sequences with the given rules.
a. $5n + 2$ b. $4b - 1$ c. $2c + 3$

Finding a general rule for a linear sequence

- A general rule for the pattern of a line sequence can be found if the working used to calculate each term is shown. **In the lines of working, determine:**

- What stays the same (the numbers that stay the same appear in the rule).
- What changes (the numbers that vary are related to the term number (n))

A general rule for a linear sequence is of the form:
$tn = an + b$, where **'n' is the term number** and **a and b are real numbers.**

A strategy (way) to find 'a' and 'b' uses the following steps.
Step 1: Identify the **difference** between each term. This is the **value of 'a'** in the rule.
Step 2: Find an amount that **must be added to or subtract from this value, 'a'** to give the first number in the sequence. This is the **value of 'b'** in the rule.

Example:
i. Find the algebraic rule for the pattern in the **sequence 6, 12, 18, 24, …**
Step 1: Identify the **difference** between each term.
6 is being added each time, so
a =6.
Step 2: The **first term** does **not require an amount** to be added to or subtracted from to make its value equal to 6, so
b = 0.

Substituting a = 6 and b = 0 in $an + b$ gives the rule $6 \times n + 0 = 6n + 0$.
So $tn = 6n$
Check: $t_3 = 6 \times 3 = 18$, as given.

ii. Find the algebraic rule for the pattern in the sequence 4, 10, 16, 22, …
Step 1: Identify the **difference** between each term. 6 is added each time so
a = 6.
Step 2: **Add or subtract an amount** to make the **first number** in the sequence,

4 = 6 – 2, so b = -2
Substituting a = 6, b = -2 in $an + b$ gives the rule 6 x n – 2 = 6n – 2.
So t_n = 6n – 2.
Check: tn = $6\ x\ 2 - 2 = 10$, as given.

Activity 10.4

1. For each of the following linear sequences:
i. Find a, the difference between each term
ii. Find b, the value added to a to make the first term
iii. Write down the rule $an + b$ for the sequence, using a and b values you found
iv. Find the 10th term in the sequence.

a. 8, 15, 22, 29, …
b. 2, 5, 8, 11, …
c. 9, 6, 3, 0, …
d. 7, 3, -1, -5, …
e. 5.5, 5.2, 4.9, 4.6, …
f. 1 500, 2 000, 2 500, 3 000, …
g. 12, 17.5, 23, 28.5, …

2. For each of the following:
i. Copy and complete the table
ii. Write an algebraic rule for the sequence
iii. Find the value of the 25th term.

a.

a	b
1	3
2	4
3	5
4	
5	

b.

a	b
1	2
2	5
3	8
4	
5	

c.

a	b
1	1
2	
3	5
4	
5	9

Finding a general rule for a linear spatial sequence

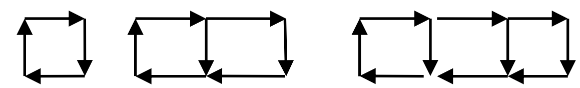
- Linear spatial sequences generate linear number sequences. This means that an **algebraic rule for a linear spatial sequence can** be found in a **similar** way to finding algebraic rule for **linear number sequences.**

Example:

A sequence of square shapes made from arrow is drawn. Find the general rule.

- The number of arrows in each shape is counted.

1 2 3



4 7 10

For every extra squares made, **another 3 arrows are added.**

Use a table to help find the pattern.

Number of squares (s)	Number pattern	Number of arrows (a)
1	$3\ x\ 1 + 1$	4
2	$3\ x\ 2 + 1$	7
3	$3\ x\ 3 + 1$	10
4	$3\ x\ 4 + 1$	13

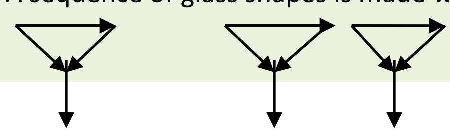
So the number of arrows needed to make 10 squares is 3 x 10 + 1 = 31.

Number of arrows = 3 x number of squares + 1.

Algebraically, **a = 3s + 1,**
where **a** = number of arrows, **s** = number of squares

Activity 10.5

1. A sequence of glass shapes is made with arrows.



1 glass **2 glasses**

a.Draw the next two patterns in the sequence.

b. Copy and complete the table.

Number of glasses	Number pattern	Number of arrows
1	4 x 1	4
2		
3		
4		

c. Write a sentence to describe the rule for the sequence.

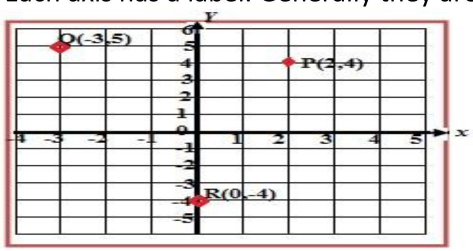
d. . Write a rule using the sentence below:
Number of matchsticks = ___ x number of glasses

e. How many matchsticks are needed to make a pattern with 20 glasses?

f. How many glasses are in a pattern with 60 matchsticks?

Graphs

- A graph shows the **relationship between two variables**. A graph is made up of two axes that **meet at the origin,** (0,0).
- Each axis has a label. Generally they are called the **x-axis and y-axis.**



Coordinates

- The position of a point on a graph is describes using a coordinate pair, made up of an **xvalue and a y-value.** The values can be negative or positive. -

The coordinates are written in brackets, **(x-value, y-value).** **The x-value is always first followed by the y-value)**

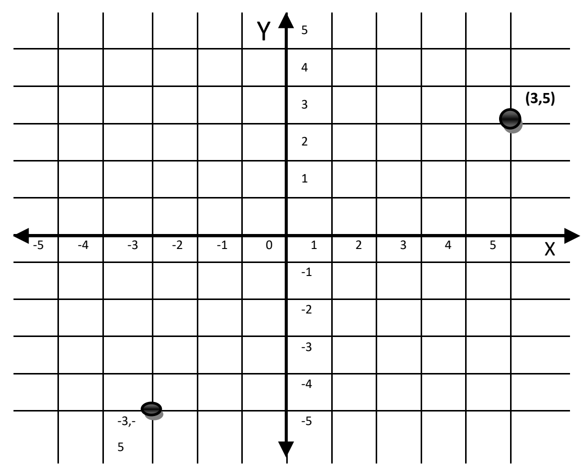
To plot a point:

i. Locate the origin (0,0)

ii. Using the first number, shift across (right for positive (+) or left for negative (-) numbers) then

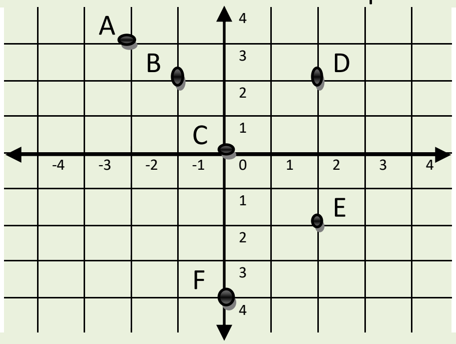
iii. Using the second number, shift up for positive (+) or down for negative (–) numbers).

Example:	
i. Plot the point (3,5).	Plot the point (-3, -5)
3 is the x-value, 5 is the y-value, So x = 3, y = 5. To plot the coordinates on the graph: i. Locate the origin (0,0) ii. Using the first number, (3) **move RIGHT 3** iii. Using the second number, (5) **move UP 5.**	-3 is the x-value, -5 is the y-value, So x = -3 and y = -5. To plot the coordinates on the graph: i. Locate the origin (0,0) ii. Using the first number, (-3) **move LEFT 3** iii. Using the second number, (-5) **move DOWN 5.**



Activity 10.6

1. Write the coordinates of the points A, B, C, D, E and F.



2. Draw a set of axes form -10 to 10 on the x-axis and -10 to 10 on the y-axis. Plot the following points and label them A to G.
A (3,5), B (2,7), C (-4,7), D (0,-5), E (-3, -6), F (-8,0), G (7,7)

3. Draw a set of axes from -5 to 5 on the x-axis and -5 to 5 on the y-axis.
a. Plot the points, A (3,2), B (5,2), C (5,0), and D(3,0).
b. Join the points in order. What shape have you made?
c. Plot the points E (-3,-2), F (-1,-2), G (-1,0), and H (-3,0).
d. Join the points in order. What shape have you made?

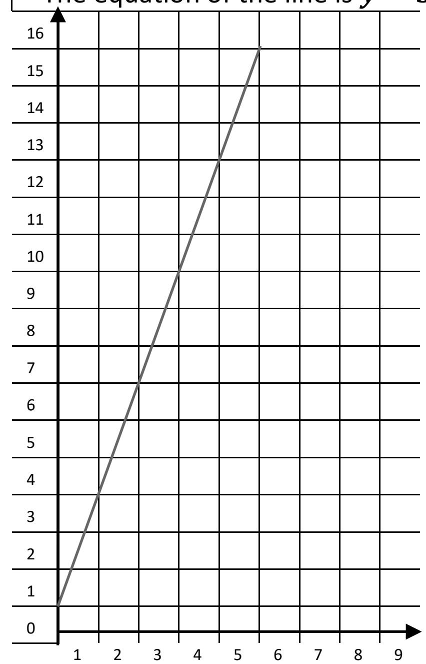
Using tables to draw a straight-line graph

Example:

- Tables of values can be drawn up for a pattern of numbers. If the rule for the pattern is $y = 3x + 1$, then the table is:

x	$y = 3x + 1$	Coordinates
1	$3 \times 1 + 1 = 4$	(1, 4)
2	$3 \times 2 + 1 = 7$	(2, 7)
3	$3 \times 3 + 1 = 10$	(3, 10)
4	$3 \times 4 + 1 = 13$	(4, 13)
5	$3 \times 5 + 1 = 16$	(5, 16)

- These coordinates can now be plotted on a set of axes.
- When the points are joined a straight line is formed. The Line can be extended. This will give **more points** whose Coordinates obey the $y = 3x + 1$.
- The equation of the line is $\boldsymbol{y = 3x + 1}$.



Activity 10.7

1) i. Copy and complete the following tables.

ii. Use the points to plot a graph.

iii. Join the points to make a straight line.

a) $y = 3x - 1$

x	y = 3x - 1	(x, y)
1	3 x 1 – 1 = 2	(1, 2)
2		
3		
4		

b) $y = 2x + 1$

x	$y = 2x + 1$	(x, y)
1	$2\ x\ 1 + 1\ 3$	(1, 3)
2		
3		
4		
5		

c) $y = 3 + 2x$

x	$y = 3 + 2x$	(x, y)
1	$3 + 2\ x\ 1 = 5$	(1, 5)
2		
3		
4		
5		